A brief history of LTB

David Brown of the SCI reviews the (relatively) recent history of lateral torsional buckling of beams. Part 1 includes a reminder of the underlying structural mechanics and the transition from theory into BS 449 and BS 5950. Part 2 looks at the comparison with BS EN 1993-1-1 and gazes into the near future.

In the beginning - Euler

Almost all buckling begins with Euler. Leonhard Euler (1707 – 1783) was a Swiss mathematician and physicist. In structural engineering he is most famous for identifying the elastic critical buckling load for a column. In the Eurocode, this load is called N_{cr} and is expressed as $N_{cr} = \frac{\pi^2 EI}{L^2}$. This is a purely theoretical load, as it assumes infinite material strength and assumes the strut is perfectly straight – neither of which is true. The obvious connection with a beam is that the compression flange is rather like a strut – if the web and tension flange are ignored.

In a beam, the resistance to lateral buckling of the compression flange is generated by:

- The lateral bending resistance of the compression flange,
- The tension flange, which restrains the compression flange, being connected by the web,
- The torsional stiffness of the section.

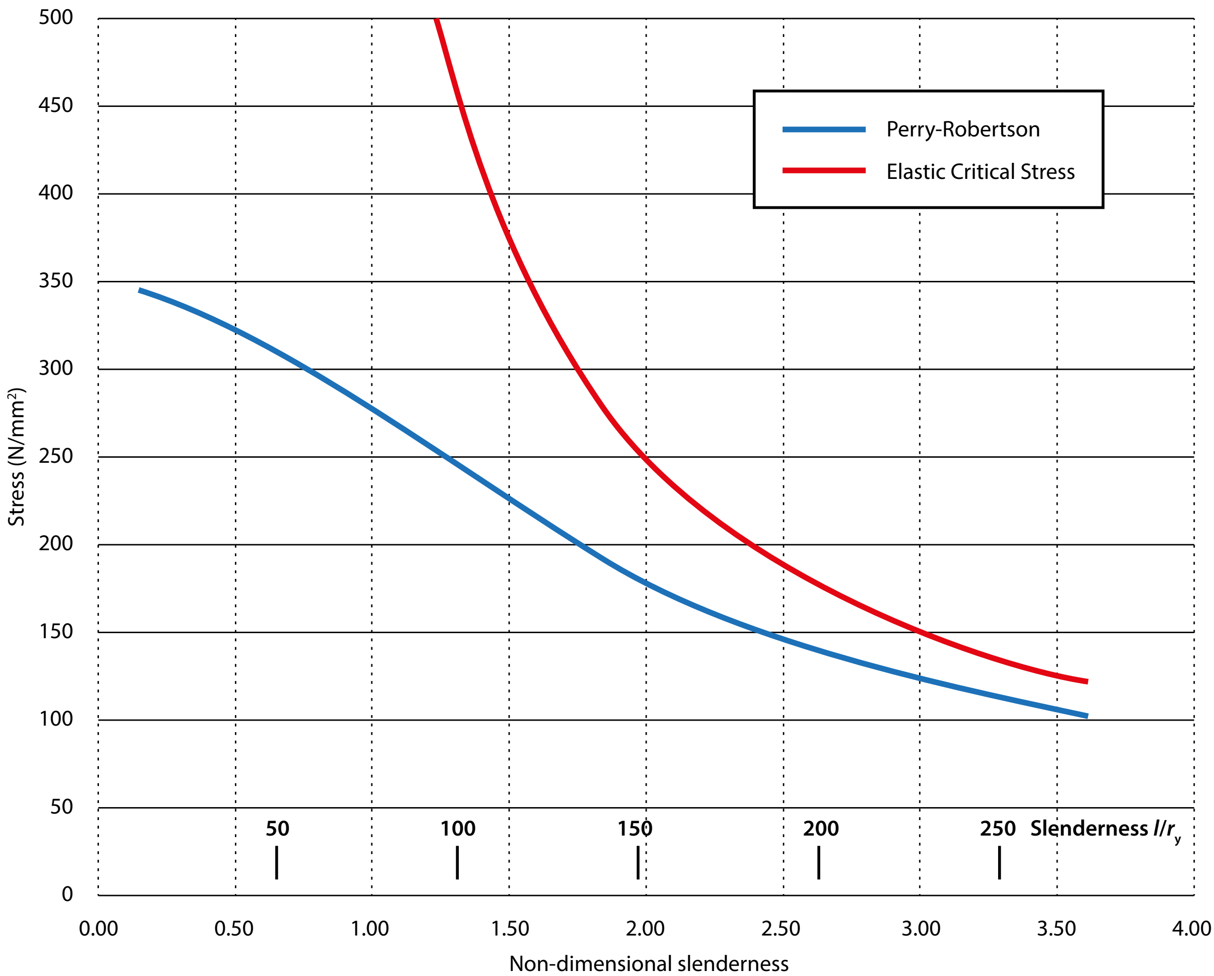
The elastic critical buckling moment for a beam is analogous to the Euler load for struts, but rather more complicated because of the additional contributions. In the Eurocode, this moment is called M_{cr}. The elastic critical stress for a beam is simply the moment divided by modulus. In the same way as a strut, the elastic critical moment is a theoretical moment, assuming infinitely strong material, and a perfectly straight beam.

From Euler to allowable stress – Messers Ayrton, Perry and Robertson

In 1886, Ayreton and Perry related the elastic critical stress to a failure stress, allowing for an initial imperfection (lack of straightness) and limited to the yield strength of the material. They did not resolve what the initial imperfections should be.

In 1925, Robertson developer the Ayrton-Perry formula, establishing imperfection values on the basis of experimental tests. This work was adopted as a basis of the strut curves (and LTB curves) in BS 449 and BS 153 (the bridge design Standard). Sadly, the reference to Ayrton seems to have been dropped and the expression became commonly known as the Perry-Robertson formula.

Figure 1
Elastic critical stress and Perry-Robertson – S355 steel



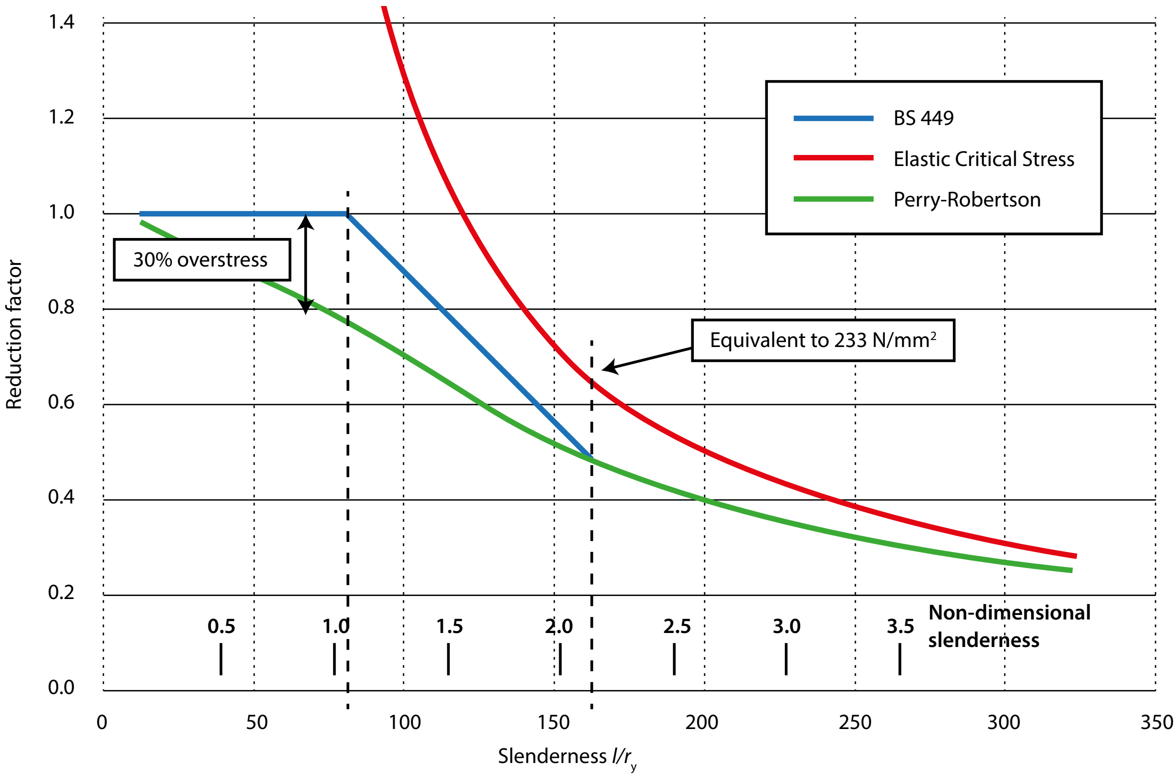


Figure 2
Normalised stresses vs slenderness

Although the precise form of the Perry-Robertson curve depends on the Perry factor assumed, Figure 1 shows the relationship between the elastic critical stress and the Perry-Robertson curve.

It should be noted that there is no plateau in Figure 1. The Perry-Robertson formula is an elastic approach and is based on failure when the stress at the extreme fibre of the section reaches yield. At low slenderness, one might expect plastic behaviour, where the whole cross section reaches yield. At low slenderness therefore, the Perry-Robertson curve is quite conservative.

Application to LTB of fabricated beams

The salient paper is by Kerensky, Flint and Brown (sadly, no relation) of 1956, where they described the basis of design for beams and plate girders in the revised bridge Standard, BS 153. This important paper was used to prepare the design guidance in the 1969 (metric) version of BS 449.

The first step is to establish the elastic critical stress in bending. Kerensky, Flint and Brown (KFB) present the critical stress for a symmetrical I section as

$$f_{\text{b,crit}} = \frac{\pi^2 EI_y h}{2Z_x L^2}\sqrt{\frac{1}{\gamma}\left\{1+\frac{4GKL^2}{\pi^2 EI_y h^2}\right\}}$$

Even without describing the variables, the comparison with the commonly-used expression for M_{cr} in the Eurocode is clear – the physics has not changed.

KFB proposed using the Perry-Robertson formula to establish an allowable stress as it had "evolved in conjunction with extensive tests and has a background of satisfactory application in design". The problem at low slenderness remained to be solved – by curve fitting. KFB proposed a plateau extending to a slenderness of l/r_y of 60, and then joining (with a straight line) to the Perry-Robertson curve at $l/r_y = 100$. KFB noted that this led to a maximum 'overstress' (compared to the Perry-Robertson stress) of 13%.

KFB recognised that for certain cross sections, the 'elastic' background to the approach could "seriously penalise" the use of such members. The problem is more noticeable when the member has a higher 'shape factor', which is $\frac{\textit{plastic modulus}}{\textit{elastic modulus}}$.

However, as they were covering plate girders, where the shape factor could be as low as 1.0, the basic formula was not modified.

Transition of KFB proposals into BS 449 for rolled sections

In BSI papers of 1969, notes are provided on the amendments to BS 449 – which included the conversion to metric units, but of more interest to this discussion, also describe the development of the LTB rules that appear in BS 449.

The basis for the BS 449 curve is the KFB paper, simplified for building designers and modified to account for the shape factor of the rolled I sections commonly used.

Firstly, the KFB formula for the critical stress is simplified. With approximations for various variables, the expression for the elastic critical stress becomes:

$$\text{Elastic critical stress} = \left(\frac{1675}{l/r_y}\right)^2\sqrt{1+\frac{1}{20}\left(\frac{lT}{r_y D}\right)^2}$$

In BS 449, this is given the symbol "A", and (if anyone can find an old copy of BS 449) appears over Table 7. In clause 20 of BS 449, this value of A is described as the elastic critical stress for girders with equal moment of inertia about the major axis – i.e. a symmetrical section. For unsymmetrical sections, the calculation of the elastic critical stress is modified.

The BS 449 drafters then dealt with the problems with the Perry-Robertson curve at low slenderness. A slightly different plateau length was proposed by extending the plateau until the Perry-Robertson stress was exceeded by the 13% described in the KFB paper, but also allowing for a shape factor of 1.15 for rolled sections. The product of these two factors is 1.13 × 1.15 = 1.3.

Thus the plateau was extended until the Perry-Robertson stress was exceeded by 30%. Although KFB proposed the intersection with the Perry-Robertson curve at $l/r_y = 100$, the drafters of BS 449 modified this to a point when the critical stress was 17/1.2 tonsf/in², or 233 N/mm². The actual slenderness at this intersection point varies with D/T.

This results in the curve (for one specific beam, with $D/T = 24$) shown in Figure 2. Note that the bending stresses have been normalised by dividing by the yield strength, to give a reduction factor. The slenderness is plotted against slenderness (l/r_y) and non-dimensional slenderness (to assist future comparisons)

The form of the BS 449 curve may be confirmed by simply plotting values in any one column from Table 3a.

Observations on the BS 449 approach to LTB

BS 449 has a simple approach to LTB. The look-up table is simple to use, but rather more complicated to embed in a spreadsheet or other program. It might also be noted that the plateau seems relatively long (The Eurocode plateau is limited to a non-dimensional slenderness of 0.4, or l/r_y = 32). Finally we note that BS 449 had no way of dealing with non-uniform moment, which was a major change introduced in BS 5950.

Bring on BS 5950

As long ago as 1969, a committee was appointed to prepare a successor to BS 449 as a limit state code. Note that the metric version of BS 449 had only just been issued!

In a background document to BS 5950, the comment is made that the new code is based on the same underlying theory as BS 449. The new rules took account of moment gradient (an improvement), but it was noted that the results of the new procedures were more conservative, especially at low slenderness. Perhaps one might expect this looking at the optimistic plateau length in Figure 2. In the background document, the elastic critical moment M_E is expressed as

$$M_E = \frac{\pi}{L}\sqrt{\frac{EI_y GJ}{\gamma}}\sqrt{1+\frac{\pi^2 EH}{L^2 GJ}}$$

, which should again look familiar.

Having calculated an elastic critical stress, BS 5950 determines an allowable bending strength using the Perry-Robertson formula, found in B.2.1 of BS 5950. The Perry factor and Robertson Constant are given. The formulation of the expressions in B.2.3 has a plateau length of λ_{LT0} .

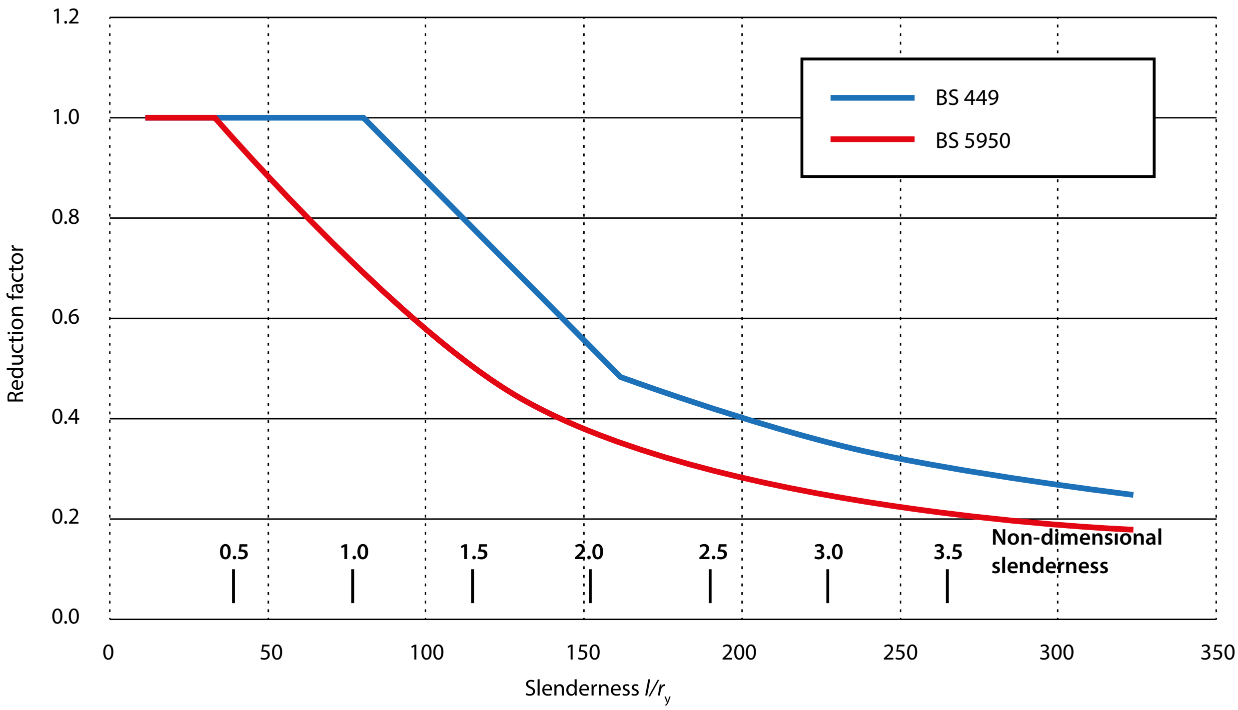


Figure 3 Comparison between BS 449 and BS 5950 LTB curves

For S355 steel, $\lambda_{LT0} = 0.4\left(\frac{\pi^2 E}{p_y}\right)^{0.5} = 30.6$

In Eurocode terms, this is equivalent to a non-dimensional slenderness of 0.38. The comparison between the LTB curves in BS 449 and BS 5950 (for a beam with *D/T* = 24) is shown in Figure 3.

The BS 5950 buckling curve is generally significantly lower than that in BS 449. Designers of a certain age may recall the general view that resistances had reduced. To some degree, this would have been offset by the change to a limit state code, when the load factor was approximately 1.55 compared to the 1.7 in BS 449. In comparisons made in 1979, it was noted that BS 449 "gives wide variations in the factor of safety" in some circumstances "which are below what is generally considered appropriate", so perhaps the reductions in resistance are not surprising.

In 1989, Amendment 8 to BS 449 was published with a revised Table 3a. For the specific beam used in this comparison, Figure 4 now shows the reduction factor as given in the revised Table. Perhaps as might be expected, the form of the curve given by Amendment 8 very closely follows that given in BS 5950. SCI has not been able to locate background documents giving the expressions behind the Amendment 8 curves – Figure 4 is simply plotted from the values in the Standard. It is not inconceivable that the Amendment follows the BS 5950 expressions, but with some allowance for the different factors of safety. If the Amendment 8 curve is plotted at 90% of its value, there is close correspondence with the BS 5950 curve – and 1.55/1.7 = 0.91. Of particular note is the much reduced plateau length compared to BS 449.

The second major change in BS 5950 was the introduction of methods to deal with a non-uniform moment, via the m_{LT} factor in Table 18. Technical exposition on the treatment of non-uniform moments appeared in AD 251 and is not repeated here.

In Part 2, the comparisons are extended to the Eurocode, with a forward-looking view of the future LTB formulae.

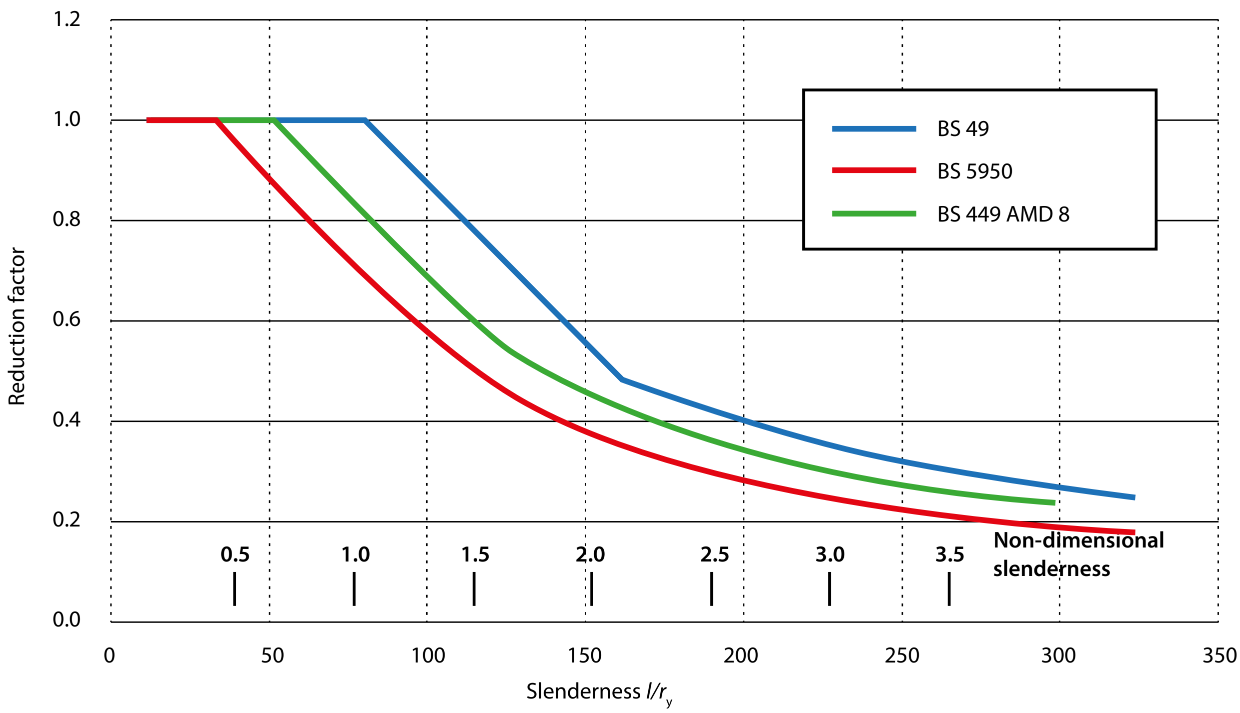


Figure 4
Comparison between BS 449, BS 5950 and BS 449 Amendment 8