

A view of Torsion – Part One

SCI's Alastair Hughes writes the first of three articles on torsion, concentrating on textbook material such as design guides.

In a steelwork designer's view, torsion is best avoided. Partly because conventional open steel sections are not very good at resisting it, but also because its analysis poses mathematical challenges that can make normal people feel rather weak. Yet it can't always be avoided, as evidenced by the popularity of SCI's P-057 which remains a best-seller 20 years after publication. The time has come for a Eurocode-aligned revision. Publication of P-385: 'Design of Steel Beams in torsion' is immanent.

What is torsion?

Just another word for twisting. Either the action of twisting (the application of Torque) or the effect (twistedness). With steelwork we are dealing with linear members, and a twisting moment is a moment about the longitudinal axis – in Eurocode terms, the x -axis. A doubly symmetrical section will twist about its centroidal axis, and will stay straight (assuming it was straight to start with, and there is no bending). The torsional displacement is manifest as rotation (φ) that varies along the length of the member. (A member that rotates bodily is not twisting.)

'Torsional moment', though really synonymous with 'twisting moment' and 'torque', is given a special meaning in P-385. In the publication, and in this article, the term is exclusively for the internal moment, an effect, symbolized T_E , whereas 'torque' is reserved for the action, T . The distinction between action and effect really matters where torsion is concerned. For a single member with equal and opposite T applied at its ends, they are numerically the same, but if the member is held at each end with T applied at midspan, each half's torsional moment T_E is equal to $T/2$. Here is a context in which time is well spent writing those little subscripts.

How is torsion resisted by steel sections?

In two very different ways. One is called 'warping', something of a misnomer but one which has become entrenched. The other is named after **St Venant** (short for Adhémar Jean Claude Barré de Saint-Venant), a French civil engineer/mathematician who developed the theory of elastic torsion and identified the phenomenon of warping. This drawing, from his 'epoch-making' memoir of 1856, depicts the pattern of warping in a square bar. We are grateful to the Radcliffe Science Library, Oxford, for this magnificent example of pre-computer draftsmanship (left).

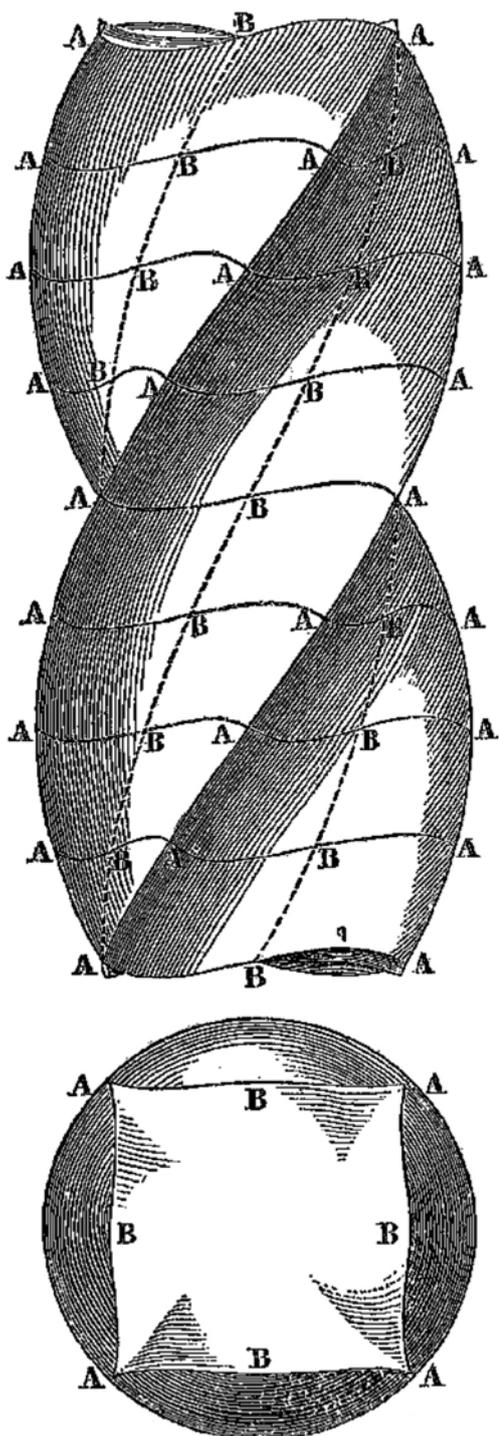
What is warping is the cross-section – in other words plane sections do not remain plane. The only sections immune to it are circular: CHS and round bars.

Having admired this drawing, it is now necessary to concede that the warping it depicts is of purely academic interest. The effects are real, but they make no significant contribution to the torsion resistance of this bar. For practical purposes, the resistance mechanism labelled 'warping' is only available to cross-sections with two flanges – such as the I-beams which are the stock-in-trade of steel framework.

In this and other respects, steelwork designers take a very selective interest in torsion theory. Our concern is for long prismatic members composed of thin elements – which can be summarized as $L \gg h \gg t$. This allows a simpler view of the subject, including the generalization just made about warping. For our purposes, so-called 'warping' is better described as **differential flange bending**. Replace torque T by a couple of equal and opposite sideways forces $\pm T/(h - t_f)$, each of which is resisted by its respective flange acting as a rectangular beam. Essentially, that is the resistance mechanism known as warping. What provides the resistance is not warping, nor (as some would have it) restraint of warping, but in-plane bending of the flanges. The warping – plane sections not remaining plane – is a side effect. The web of an I-beam, astride the neutral axis of both flanges, is disengaged, and may as well be ignored in 'warping' calculations.

Of course the flanges can only display resistance if they have something to react against, so differential flange bending is not available to an isolated bar which is picked up and twisted from end to end, but in most practical situations it can act.

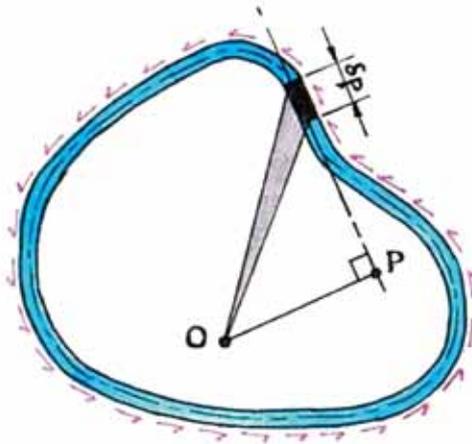
It is quite legitimate, with two parallel flanges available, to disregard St Venant and



verify torsion resistance in the manner just described, using familiar bending theory without reaching for a specialist Design Guide. Conversely, if two parallel flanges are **not** available (e.g. angle, cruciform and T-sections) ‘warping’ can and should be disregarded – it has virtually nothing to contribute.

St Venant resistance

That leaves the other resistance mechanism. In some textbooks, St Venant is all there is to torsion. It is variously described as ‘pure’ or ‘uniform’ torsion, but both these labels leave something to be desired and the choice of ‘St Venant’ for the Eurocodes is to be applauded.



The simplest explanation of the way it works is to consider a thin-walled hollow section (of any shape, see the diagram above). The torsional moment T_E is resisted by a shear force which flows around the perimeter. Suppose the wall thickness is uniformly t and the shear stress is τ , which is assumed to be constant as the wall is thin. Each element of perimeter δp , such as the one arbitrarily picked out on the sketch, provides a force $\tau \delta p$ acting with a lever arm OP about an arbitrary axis O . The moment it exerts is equal to τt times twice the area of the shaded triangle. Since τt is constant around the perimeter, it follows that the total torsional moment resistance is equal to τt times twice the area enclosed by the mean perimeter A_p , and the torsional section modulus $W_t (= T_E/\tau)$ is $2tA_p$.

Let us try this out on a 508 x 20 CHS: $W_t = 2 \times \pi \times 0.244^2 \times 0.02 = 7.48E-3 \text{ m}^3$, a reasonable approximation to the ‘exact’ figure of 7200cm^3 in the Tata Steel Tubes handbook. Incidentally, W_t is what used to be symbolized C and described as ‘torsional modulus constant’. ‘St Venant torsional section modulus’ seems a more helpful way to describe it.

The other important section property is the ‘St Venant torsional constant’, traditionally symbolized J but now to be referred to as I_t .

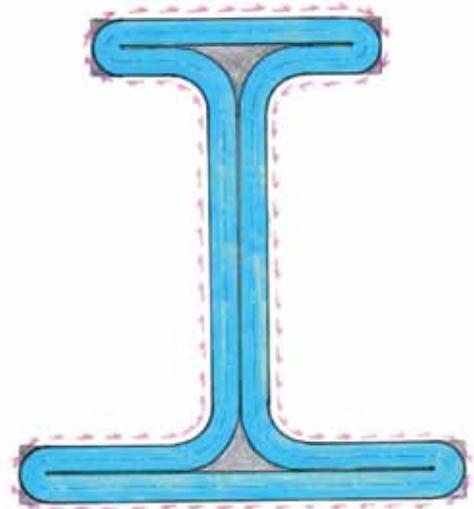
$G I_t$ is the St Venant torsional stiffness, analogous to bending stiffness or flexural rigidity EI . Just as EI is the slope of the graph of bending moment versus curvature, $G I_t$ is the slope of the graph of torsional moment versus ϕ' , the first derivative of ϕ with respect to x . Over length L the member will rotate by $T_E L/G I_t$ (assuming constant torsional moment and no warping).

To estimate I_t for the thin-walled hollow section, consider unit length of member rotating $T_E/G I_t$. The work done per unit length is $T_E^2/G I_t$. The entire section is stressed to τ , so strain energy per unit volume is τ^2/G . Per unit length this is $\tau^2 p t/G$ where p is the length of the mean perimeter. Equating the two, and substituting $2tA_p \tau$ for T_E , we get $I_t = 4 A_p^2 t/p$.

Again, let us try this out on 508 x 20 CHS: $I_t = 4 \times (\pi \times 0.244^2)^2 \times 0.02/(\pi \times 0.488) = 1.825E-3 \text{ m}^4$. Not a bad approximation; the handbook value is $183000 \text{ cm}^4 (1.83E-3 \text{ m}^4)$.

To show that there is no fundamental difference, try the

formula on a (carefully chosen) I-section. Here a hollow section of uniform thickness $t = 20 \text{ mm}$ is squashed into the form of 300 ASB 249, which happens to have equal thickness ($T' = 40 \text{ mm}$) all round:



The area enclosed by the mean perimeter can be calculated as $[(b_t - T) + (b_b - T) + (h - 1.5T)]T/2 + 2\pi(T/4)^2 + (4 - \pi)(r + T/4)^2 = 16.16E-3 \text{ m}^2$, and the mean perimeter is $2[(b_t - T) + (b_b - T) + (T + 2r) + (h - 2T - 2r)] + 2\pi(3T/4 + r) = 1.458 \text{ m}$.

$I_t = 4 A_p^2 t/p = 4 \times (16.16E-3)^2 \times 0.02/1.458 = 13.5E-6 \text{ m}^4$, which compares with $2004 \text{ cm}^4 (20E-6 \text{ m}^4)$ in the handbook.

While the result is in the right order, the comparison is nothing like as good, as the thin-walled hollow section assumption has been stretched too far. In this case it would imply an abrupt change in the direction, but not the magnitude, of the shear stress at mid-thickness. So long as it remains within the elastic range, the St Venant shear stress in a parallel sided element will actually increase linearly from zero at mid-thickness to a peak at each surface.

St Venant torsion in typical I-sections, whose webs are thinner than their flanges, is still more complicated and will be the subject of Part Two.

But first imagine unfolding the squashed section into a 484 mm diameter CHS. It would become $4(\pi \times 0.232^2)^2 \times 0.02/1.458 = 1.57E-3 \text{ m}^4$, which is getting on for **eighty** times as stiff. The superiority of the hollow section in terms of strength is also immense, though not as simple to compare. In part, that is because conventional sections are relatively likely – indeed almost certain – to reach a limit state of excessive deformation long before their resistance is exhausted.

Where torsion is at large, there is a compelling case for checking serviceability ahead of strength, advice which applies with hollow sections as well as open ones. Not for nothing is torsion a favourite of suspension designers looking to absorb elastic strain energy.

Serviceability checking involves comparison of calculated displacement – in this context, rotation – with a subjectively set limit of acceptability. The Eurocode and NA provide no quantitative guidance, though P-385 offers some tentative advice. Serviceability limits are, properly, a matter of case-by-case designer judgement.

It is appropriate to end Part One with three mantras:

- Try to avoid torsion by lining beams up with their load
 - If serious torsion cannot be designed out, preferably resist it with a hollow section
 - Don't let torsionally induced displacements get out of hand
- In Part Two: insight into St Venant, by means of the membrane analogy.