Calculating the C_1 factor for lateral torsional buckling

David Brown of the SCI proposes expressions for C_1 for use with linear and non-linear bending moment diagrams

Background

When calculating the lateral torsional buckling resistance of an unrestrained member, the shape of the bending moment diagram is important. A uniform bending moment diagram is the most onerous, and non-uniform bending moment diagrams less onerous.

In BS 5950, the influence of the shape of the bending moment diagram is allowed for using the equivalent uniform moment factor, m_{LT}. In BS EN 1993-1-1, the shape of the bending moment diagram influences the calculation of the elastic critical buckling moment, M_{cr}, which is a step on the way to calculate the slenderness of the member.

Calculation of M_{cr}

The elastic critical buckling moment, M_{cr} may be calculated using software (notably *LTBeam*[1] and, shortly, a tool to be released on *Steelconstruction.info*) or by an expression. Within the software, the shape of the bending moment diagram follows from the loading which is input. If using software, it is educational to change all the loads by the same proportion and see that the value of M_{cr} does not change; the shape of the bending moment diagram is important, not the size, in determining M_{cr}.

If calculating M_{cr} using an equation, the equation is of the form $M_{cr} = C_1 \times$ (expression), where C_1 depends on the shape of the bending moment diagram. C_1 may be obtained from sources of non-contradictory complementary information (NCCI), such as the *Concise Guide*[2], or found on *Steelbiz*.

General expressions for C_1

Designers frequently request a general expression for C_1, so that it can be calculated automatically, (for example in software, or a spread sheet) rather than referring to tabular NCCI. Expressions for C_1 certainly exist from various sources, but are inconsistent, especially for non-linear bending moment diagrams. This article sets out the expressions that SCI generally uses, though others certainly could be used.

When assessing the accuracy of general expressions for C_1, the UK National Annex defines C_1 in clause NA.2.18 as:

$$C_1 = \frac{M_{cr} \text{ for the actual bending moment diagram}}{M_{cr} \text{ for a uniform bending moment diagram}}$$

Of course, to use this expression in the first instance, M_{cr} needs to be known, which depends on C_1, so in any comparisons made, SCI have tended to use software to calculate M_{cr}, to then compute C_1 for given shapes of bending moment diagram.

Linear bending moment diagrams

Although different expressions can be found, SCI generally uses the expression:

$$C_1 = 1.77 - 0.88\psi + 0.11\psi^2$$

Where ψ is the ratio of end moments.

This expression follows from the UK National annex where

$$k_c = \frac{1}{\sqrt{C_1}}$$

The NA is anticipating that k_c will be calculated based on C_1, but if the logic is reversed (a potential source of error), C_1 may be calculated for given values of k_c. Table 6.6 of BS EN 1993-1-1 has an expression for k_c for linear bending moment diagrams:

$$k_c = \frac{1}{1.33 - 0.33\psi}$$

Substituting and rearranging leads to the above expression for C_1.

A common expression found in several other sources, notably ECCS guides[3], is

$$C_1 = 1.77 - 1.04\psi + 0.27\psi^2, \text{ but} \leq 2.6$$

There is little difference in the outcome. Figure 1 compares the "SCI curve" against the above expression (described as the "ECCS curve"); the difference is only a few per cent. *LTBeam* can also be used to determine the value of C_1. The comparison is also shown in Figure 1.

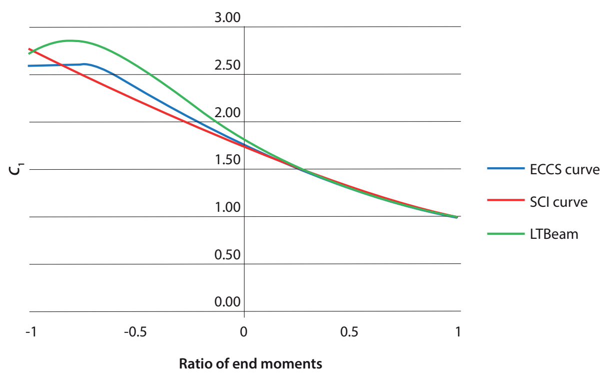

Figure 1: Comparison of C_1 curves

For positive ratios of end moments, the three curves are almost superimposed. For negative ratios, (a reversing bending moment diagram) the difference is more pronounced. Although the curve preferred by the SCI is slightly conservative, it does have the merit of having some provenance from the Eurocode itself.

Non-linear bending moment diagrams

There are many expressions proposed for C_1 for a non-linear bending moment diagram, in various research papers, but a general expression that shows good correspondence across all shapes of bending moment diagram remains elusive. In 2011, SCI reviewed the published expressions and tentatively suggested that the expression proposed by Serna et al[4] had a reasonable balance of accuracy and useability. For the common case of fork end supports (both lateral bending and warping free), the proposed expression for C_1 becomes

$$C_1 = \sqrt{\frac{35M_{max}^2}{M_{max}^2 + 9M_2^2 + 16M_3^2 + 9M_4^2}}$$

Where the values of M are shown in Figure 2. Readers will appreciate the conceptual similarity with the general expression for m_{LT} in Table 18 of BS 5950.

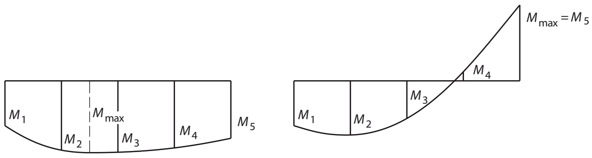

Figure 2: Values of M

Example 1

6m beam with 40 kN/m, and end moments of -50 and 200 kNm

The resulting bending moment diagram is shown in Figure 3

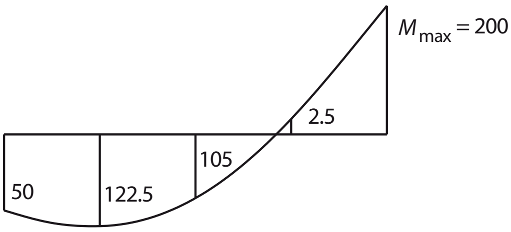

Figure 3: Bending moment diagram for Example 1 (kNm)

$$C_1 = \sqrt{\frac{35 \times 200^2}{200^2 + 9{\times}122.5^2 + 16{\times}105^2 + 9{\times}2.5^2}} = 2.0$$

Using *LTBeam* the value of C_1 is computed as 2.03

Example 2

6m beam with 40 kN/m, and end moments of -150 and 50 kNm

The resulting bending moment diagram is shown in Figure 4

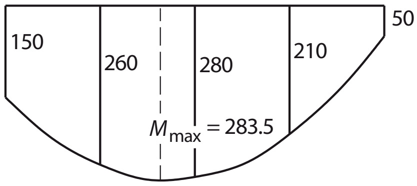

Figure 4: Bending moment diagram for Example 2 (kNm)

$$C_1 = \sqrt{\frac{35 \times 283.5^2}{283.5^2 + 9{\times}260^2 + 16{\times}280^2 + 9{\times}210^2}} = 1.1$$

Using *LTBeam* the value of C_1 is computed as 1.1

Conclusions

Closed expressions are available as an alternative to using software to determine C_1 and are reasonably accurate for standard (simple) end conditions. With other end conditions, software is the best approach.

[1] LTBeam
Available from *http://www.cticm.com/*

[2] Brettle, M.E; Brown, D. G
Steel Building Design; Concise Eurocodes (P362)
SCI, 2009

[3] Boissonnade, N; Greiner, R; Jaspart, J. P; Lindner, J.
Rules for Member Stability on EN 1993-1-1
Background documentation and design guidelines
ECCS guide No. 119
ECCS, 2006

[4] Serna, M. A; Lopez, A; Puente, I; Yong, D
Equivalent uniform moment factors for lateral-torsional buckling of steel members
Journal of Constructional Steel Research, Volume 62, 2006
Elsevier, 2006

New and revised codes & standards

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ISO 148-3

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CEN EUROPEAN STANDARDS

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Eurocode 8. Design of structures for earthquake resistance. Assessment and retrofitting of buildings
CORRIGENDUM 2: August 2013 to EN 1998-3:2005