How Concrete Shrinkage Affects Composite Steel Beams

Stuart Alexander

Concrete shrinks. Not much, but enough to warrant thinking about in design. Steel doesn't, so in composite construction shrinkage of insitu concrete slabs induces stresses and deflections in the supporting steel beams.

Shrinkage can be important for two aspects of serviceability – stress limitation and deflection. This article explains the theory and shows how to calculate the effects. Numerical examples are given, and the importance of concrete shrinkage is assessed. For simplicity, the article is limited to simply supported beams in buildings in indoor conditions.

Derivation of contraction force

Consider a concrete slab cast insitu on metal decking on a steel beam, and firstly allow the contraction of the slab to take place freely, as if the interface is greased and ignoring the shear connectors (figs 1a, 1b). Then apply a force at the centroid of the slab with an equal and opposite reaction on the beam so that the relative movement at the interface is eliminated (fig. 1c); this models the assumption that there is no slip between the slab and the beam at service loads.
By defining the strain in the slab as $\boldsymbol{\varepsilon_r}$, the force $\mathbf{F = \varepsilon_r A_c E_c}$. Applying the equal and opposite force to the beam produces a strain at the level of the centroid of the slab given by:

$$\varepsilon_n - \varepsilon_r = F\,\{(1/E_sA_s) + (z^2/E_sI_s)\}.$$

Eliminating ε_r gives:

$$\begin{aligned} F &= \varepsilon_n / \{(1/E_cA_c) + (1/E_sA_s) + (z^2/E_sI_s)\} \\ &= \varepsilon_n E_s I_s / \{(m\,I_s/A_c) + (I_s/A_s) + (z^2)\}. \end{aligned}$$

In this, **A**, **E** and **I** are used conventionally, suffixes **c** and **s** refer to the concrete slab and the steel beam respectively, **z** is the lever arm between the centroids of the slab and the beam and m is the modular ratio $\mathbf{E_s/E_c}$. Note that $\mathbf{A_c}$ is the average cross-sectional area of the slab, not the minimum as used in strength calculations. $\boldsymbol{\varepsilon_n}$ is the net contraction, i.e. after allowing for the restraint of both the reinforcement and the metal decking; this is explained below.

This expression can be clarified by writing $\mathbf{m\,I_s/A_c = q^2}$ and $\mathbf{I_s/A_s = r^2}$**;** **q** is a measure of influence of the slab on the beam and **r** is the radius of gyration. Then:

$$F = \varepsilon_n E_s I_s / (q^2 + r^2 + z^2).$$

Sources of contraction

There are two potential sources of contraction, early thermal and shrinkage. Early thermal contraction is caused when fresh concrete, which is heated by the chemical action of hydration, hardens and cools down. However, in composite slabs the heat of hydration can escape from both the top and bottom surfaces of what is a relatively thin member and so the temperature rise will be very small, a few degrees at most; early thermal contraction can therefore be ignored.
More important is shrinkage, strictly drying shrinkage. In *Understanding shrinkage and its effects* (ref. 1) the author gives data for estimating the free shrinkage $\boldsymbol{\varepsilon_{cs}}$; this combines the guidance in BS 5400-4 (ref. 2) and BS 8110-2 (ref. 3). He also shows that the effect of reinforcement is to reduce the contraction so that:

$$\varepsilon_n = e_{cs} / (1 + m\,\rho),$$

where ρ is the ratio of reinforcement area to concrete area.

At this point some assumptions need to be clarified. The first is that because the metal decking is bonded continuously to the concrete, it can be treated in the same way as embedded reinforcement. And because the slab is attached to the steel beam which forces it to contract linearly it does not matter whether the decking - or for that matter the reinforcement - is not concentric in the section. For typical slab profiles with steel decking 0.9-1.2 mm thick and A142 fabric reinforcement, ρ ranges from 1.0 to 1.4%, i.e. much higher than is normal in conventional reinforced concrete.
The author (ref. 1) goes on to show that although the shrinkage occurs continuously, and each increment is subsequently relieved by creep, this can be modelled by applying the total shrinkage in one step at age 150 days. This gives a typical value of the modular ratio **m** of 17.5. Thus the effect of the restraint is a factor of 0.80 - 0.85.

Effects of contraction force

The force **F** is then applied as an external force to the steel beam (not to the composite section) enabling any property to be calculated; this can include extreme fibre stresses as well as the curvature from which the deflection is then calculated.

Extreme fibre stresses

The top (compression) and bottom (tension) extreme fibre stresses $\mathbf{f_{st}}$ and $\mathbf{f_{sb}}$ respectively are given by:

$$f_{st}, f_{sb} = \varepsilon_n E_s (r^2 \pm Dz/2) / (q^2 + r^2 + z^2)$$

where **D** is the depth of the steel beam.

Curvature and deflection

Curvature κ is given by **M/EI**, so here:

$$\kappa = F\,z / E_s I_s = \varepsilon_n z / (q^2 + r^2 + z^2).$$

The deflection δ is then given by:

$$\delta = 0.125\,\kappa\,L^2$$

where **L** is the span, assumed simply-supported, i.e. ignoring any continuity at the supports.
A different formula for curvature is given in the *Steel Designers' Manual* (ref.4) and repeated in *Composite beam design to Eurocode 4* (ref.5); the derivation is not explained. This replaces the term $\mathbf{q^2 + r^2 + z^2}$ with

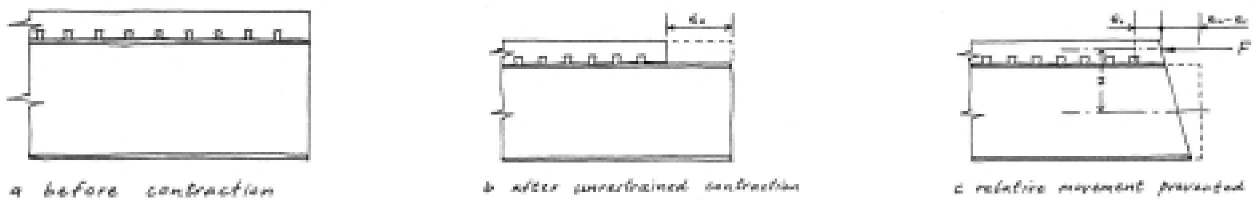

Fig. 1. Contraction of insitu concrete topping.

$(1 + \mathbf{m}\ \mathbf{A_s} / \mathbf{A_c})\ \mathbf{I_{cs}} / \mathbf{A_s}$, where $\mathbf{I_{cs}}$ is the **I** of the composite section assuming the concrete is uncracked. This gives lower curvatures and deflections; the reduction is virtually nil where the slab is comparable to the beam (ratio $\mathbf{m}\ \mathbf{A_s} / \mathbf{A_c}$ around 1), but is down to over one-half when the slab dominates (ratio $\mathbf{m}\ \mathbf{A_s} / \mathbf{A_c}$ around 0.3).

Practical application

To show the effect of the calculation, a numerical example is presented in table 1. A typical value of shrinkage of 400 με (micro-strain, i.e. strain x 10^{-6}) for internal conditions is adopted, reducing to say 325 με after allowing for restraint. A 2.5 m wide slab of overall depth 130mm on re-entrant profile decking 51 mm deep using grade C30 concrete on a range of beam sections has been examined. Three beam depths have been taken, with the lightest and heaviest rolling in each. Deflections are based on the maximum spans from table 2 of *Design of composite slabs and beams with steel decking* (ref.6) for imposed loading 4.0 kN/m². The relatively small variation between widely different steel beams is interesting. The deflections are all somewhat less than span/750.

Codes and standards

What references are there in codes and standards? The principal current guide is the code of practice for design of composite beams BS 5950: Part 3: Section 3.1 (ref.7) ('BS 5950-3-1'). This gives rules for calculating deflections under 'serviceability loads' (defined as the unfactored values) including an addition for partial shear connection. It also recognises the risk of 'irreversible deformation' under 'normal service conditions' and limits the stress in the extreme fibre to the design strength $\mathbf{p_y}$; any effect of partial shear connection is excluded. However, shrinkage of the concrete slab is not mentioned as a contribution to service conditions for either deflection or stress limitation.

BS 5950-3-1 will eventually be supplanted by Eurocode 4 (ref.8), currently available in draft. This states that "calculation of stresses and deformations at the serviceability limit state shall take into account the effects of [*inter alia*] creep and shrinkage of concrete", and further that "the effect of curvature due to shrinkage of concrete should be included when the ratio of span to overall depth of the beam exceeds 20 and the predicted free shrinkage strain of the concrete exceeds 400 x 10^{-6}". The effect on stress is not specifically mentioned.

Are these stresses and deflections significant?

Stress limitation

The underlying assumption of design for the ultimate limit state is that as the load is gradually increased each part in tension or compression successively yields until the whole of the section is at the yield stress. So as the load is increased any pre-existing stresses – both compressive and tensile – are over-ridden and cease to be significant. However, the same is not true for serviceability behaviour. In principle, stresses under service loading should not exceed the design stress $\mathbf{p}_y$ so as to prevent incremental irreversible deformation; this principle is called "stress limitation".

Although not explicit in the codes, overstress in the top flange is not important as, if it occurs, the force is simply transferred from the steel beam to the concrete slab with minimal deformation. However, the same is not true of the bottom flange; the tensile stress is around 10-15% of the design stress and should perhaps be included in the assessment of stress under service loading. However, invariably the service load is over-estimated (see the author's *Imposed floor loading for offices: a re-appraisal* (ref. 9)) and the design stress is below the onset of yield, so ignoring the tensile stress induced by shrinkage is acceptable.

Deflection

The shrinkage deflection is clearly significant, and should be included as part of the total long-term deflection. Interestingly, shrinkage acts on the shear connectors in the opposite direction to applied loading, which suggests that allowing for partial shear connection is over-conservative. A simple rule would be to assume shrinkage deflection is equal to span/750 unless it is estimated by a more accurate calculation – in spite of the silence in BS 5950-3-1 and the let-off in Eurocode 4.

Stuart Alexander is Group Technical Co-ordinator at WSP Group.

Table 1. Long-term stresses (in N/mm²) and deflections (in mm and as L/xxx) generated by typical shrinkage (net 325 με) in a composite concrete slab.

Beam section	356 x 127 x 33 UB	356 x 171 x 67 UB	457 x 152 x 52 UB	457 x 191 x 98 UB	533 x 210 x 82 UB	533 x 210 x 122 UB
Top fibre compression (N/mm²)	77	70	73	65	67	61
Bottom fibre tension (N/mm²)	-41	-35	-35	-29	-30	-26
q (mm)	75	115	120	176	179	227
r (mm)	140	151	179	191	213	221
z (mm)	253	260	304	312	343	351
Maximum span L (m)	8.9	11.8	12.2	13.9	14.2	15.4
Simply-supported deflection	9 mm L/980	14 mm L/830	13 mm L/920	15 mm L/940	14 mm L/990	15 mm L/1020

The relatively small variation between widely different steel beams is interesting. The deflections are all somewhat less than span/750.

References

1. Alexander S.J., "Understanding shrinkage and its effects"; *Concrete*, October 2002; pp 61-63; and November/December 2002; pp 38-41.
2. BSI; *BS 5400 Steel, concrete and composite bridges, Part 4 Code of practice for design of concrete bridges; BS 5400: Part 4: 1990.*
3. BSI; *BS 8110: Structural use of concrete, Part 2: Code of practice for special circumstances; BS 8110: Part 2: 1985.*
4. Owens G.W. and Knowles P.R. (ed.), *Steel Designers' Manual*, 5th edition; Blackwell; 1992.
5. Lawson R.M. and Chung K.F.; *Composite Beam Design to Eurocode 4*; Steel Construction Institute publication 121; 1994.
6. Lawson R.M., *Design of composite slabs and beams with steel decking*; Steel Construction Institute publication 055; 1989.
7. BSI; *BS 5950: Structural use of steelwork in building, Part 3: Design in composite construction, Section 3.1 Code of practice for design of simple and continuous composite beams; BS 5950: Part 3: Section 3.1: 1990.*
8. BSI; *Eurocode 4: Design of composite steel and concrete structures, Part 1.1: General rules and rules for buildings, DD ENV 1994-1-1, 1994.*
9. Alexander S.J.; "Imposed floor loading for offices: a re-appraisal", *The Structural Engineer*; 3 December 2002; pp 35-45.