The design of hybrid fabricated girders

David Brown of the SCI discusses the design of hybrid fabricated girders. In the first part of the article, some background is presented, and a worked example taken as far as the moment resistance. Shear resistance will be covered in Part 2.

Why hybrid?

Hybrid girders are plate girders with flanges of higher strength than the web. Conceptually, one might say that the web merely keeps the flanges apart, so why not use a lower steel strength for the web? The web must carry the shear force, but this is generally low in a beam designed for bending or deflection, so high strength webs are not required. The low demand for shear resistance coupled with the desire to keep the flanges far apart means that webs in fabricated plate girders are often deep and thin – making a stiffened web likely and triggering a visit to BS EN 1993-1-5 to determine properties for Class 4 sections.

Shear Lag

Shear lag may affect both compression and tension flanges. Ordinarily, it is assumed that the stress distribution across a flange is uniform, as shown in Figure 1. In fact, the longitudinal stresses are transmitted through the web-to-flange junction. It may readily be imagined that the flange local to the web is compressed more than the flange tips, as indicated in Figure 2. The tips of the flanges "lag" in that they do not take the assumed evenly distributed share of stress. The phenomena is managed in BS EN 1993-1-5 by calculating a reduced effective width of the flange.

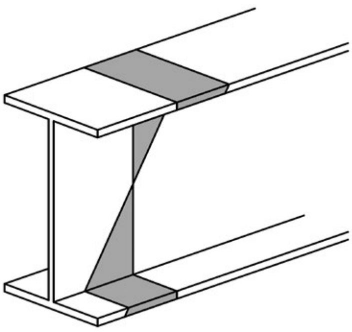

Figure 1: Commonly assumed stress profile

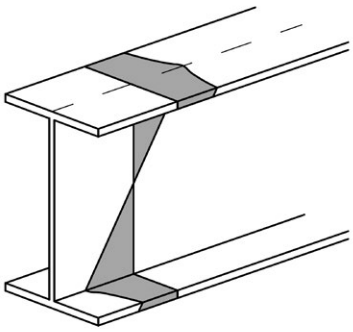

Figure 2: Shear lag in flanges

Plate buckling - flanges

All elements in compression share an enthusiasm to buckle – so the tips of relatively thin, wide flanges wish to buckle locally and do not carry load as assumed. Plate buckling only applies to the compression flange and is managed in BS EN 1993-1-5 by reducing the effective area of the flange.

Plate buckling – web

The compression zone of a thin web will suffer from local buckling. This is managed by a "hole in the web" approach where the ineffective portion of the web is neglected. The Standard specifies the stable lengths of web attached to the flange and attached to the tension zone of the web.

Stress distribution

Combining a lower strength web with higher strength flanges and assuming an ineffective portion of the compression zone of the web, the resulting stress distribution may look something like that shown in Figure 3. It is not possible to determine a modulus directly, so the position of the neutral axis is found by equating the tension to the compression. The hole in the web adds complication to the process, because the stable parts are a proportion of the compression zone – and therefore change as the neutral axis moves. Others designers may have a clever way to determine when equilibrium of force is reached – the SCI approach is to move the neutral axis a (small) step at a time, check the resulting forces, and repeat as necessary until the solution is found. This is a task for a spreadsheet, or VBA.

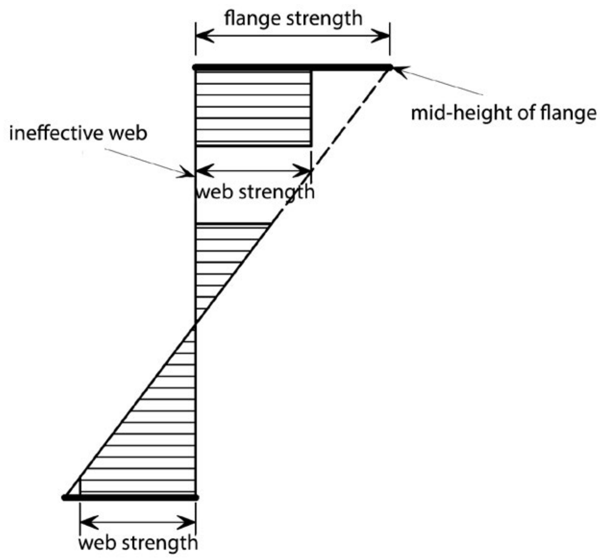

Figure 3: Typical stress profile for a hybrid girder

Once the stress distribution has been determined, the moment resistance of the cross section $M_{cy,Rd}$ may be calculated, being the product of stress, area and lever arm.

Worked example

The cross section to be verified is shown in Figure 4 overleaf. The flanges are S460 and the web S355. The beam span is 8 m.

Material strengths (BS EN 1993-1-1) and classification

Because the flange is greater than 16 mm, $f_y = 440$ N/mm^2

$$\varepsilon = \sqrt{\frac{235}{440}} = 0.73$$

$$\text{Flange outstand} = \frac{400-12}{2} = 194 \text{ mm}$$

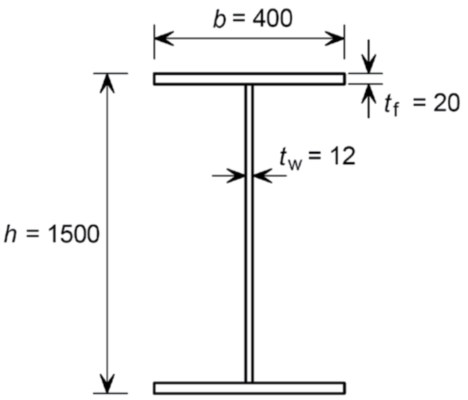

Figure 4: Cross section dimensions

$c/t = 194/20 = 9.7$
Class 2 limit is $10\varepsilon = 7.3$
Class 3 limit is $14\varepsilon = 10.2$, so the flange is Class 3
for the web, $\varepsilon = 0.81$
$c/t = 1460/12 = 121.7$
Class 3 limit is $124\varepsilon = 100.9$, so the web, and therefore the section, is Class 4.

Shear Lag (clause 3.1(1) of BS EN 1993-1-5)

$b_0 = 400/2 = 200$. Note in Figure 3.2 of the Standard, b_0 is half the flange width.
$L_e/b_0 = 8000/200 = 40$. As this is not greater than 50, shear lag cannot be neglected.
From Table 3.1, because there are no longitudinal stiffeners, $A_{sl} = 0$ and therefore $\alpha_0 = 1.0$
$\kappa = \alpha_0 b_0/L_e = 1 \times 200/8000 = 0.025$
because $0.02 < \kappa \le 0.7$, and there is a sagging bending moment diagram:

$$\beta = \beta_1 = \frac{1}{1+6.4\kappa^2} = \frac{1}{1+6.4 \times 0.025^2} = 0.993$$

Stress distribution due to shear lag (clause 3.2.2 of BS EN 1993-1-5)

Because $\beta > 0.2$, the stress distribution is shown in Figure 5.
The ratio of stresses is needed later, so the calculation is best expressed as:

$$\frac{\sigma_2}{\sigma_1} = 1.25(\beta - 0.2) = 1.25(0.996 - 0.2) = 0.995$$

The value of 0.995 indicates that there is hardly any influence from shear lag in this example.

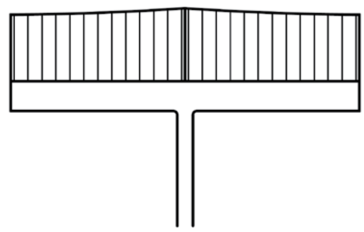

Figure 5: Stress distribution across flange outstand

Flange plate buckling (Clause 4.4 of BS EN 1993-1-5)

From Table 4.2, for outstand elements, $\psi = \dfrac{\sigma_2}{\sigma_1} = 0.995$

therefore $k_\sigma = \dfrac{0.578}{\psi + 0.34} = \dfrac{0.578}{(0.995+0.34)} = 0.433$

then $\bar{\lambda}_p$ is given by clause 4.4(2) as:

$$\bar{\lambda}_p = \frac{\bar{b}/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{194/20}{28.4 \times 0.73 \times \sqrt{0.433}} = 0.711$$

note that $\bar{b} = c$ for outstand flanges. $c = (400 - 12)/2 = 194$ mm
because $\bar{\lambda}_p < 0.748$, $\rho = 1.0$
effective^p area $A_{c.eff} = 1.0$ Ac $= 400 \times 20 = 8000$ mm^2
The superscript ^{p} indicates this is the effective area when considering plate buckling.

Combined effects of shear lag and buckling (clause 3.3(1), Note 3, of BS EN 1993-1-5)

The effective area of the compression flange considering both shear lag and plate buckling is given by:
$A_{eff} = A_{c,eff}\beta^\kappa = 8000 \times 0.996^{0.025} = 7999$ mm^2
There is therefore no reduction due to the effects of shear lag and plate buckling.

Web buckling

Because (in this case) there is no reduction of the compression flange due to the combined effects of shear lag and plate buckling, and no reduction of the tension flange due to shear lag, the gross cross section is symmetrical. The neutral axis of the gross section is at mid-height of the web.
The length of the compression part of the web b_c is $1460/2 = 730$ mm
Because the gross cross section is symmetrical, $\psi = -1$, and from Table 4.1 of BS EN 1993-1-5, $k_\sigma = 23.9$
According to clause 4.3(6), the yield strength of the flange must be used when determining the effective area of the web. Because

$f_{yf} = 440$ N/mm^2, $\varepsilon = 0.73$. Then

$$\bar{\lambda}_p = \frac{\bar{b}/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{1462/12}{28.4 \times 0.73 \times \sqrt{23.9}} = 1.20$$

$$0.5+\sqrt{0.085-0.05\psi} = 0.5+\sqrt{0.085-0.55\times(-1)} = 0.874$$

$$\bar{\lambda}_p = 0.874\text{, so } \rho = \frac{\bar{\lambda}_p - 0.055(3+\psi)}{\bar{\lambda}_p^{\,2}} = \frac{1.2-0.055\,(3+(-1))}{1.2^2} = 0.757$$

The effective depth of the compression part of the web is therefore $\rho \times b_c = 0.757 \times 730 = 553$ mm
From Table 4.1, the stable length adjacent the compression flange is $0.4b_{eff} = 0.4 \times 533 = 221$ mm
The ineffective length (the 'hole') = 730 – 533 = 177 mm
According to clause 4.3(5) the stress in the flange is considered at the mid-plane of the flange.

Stress Block

By postulating a position of the neutral axis, the stresses at locations throughout the cross section can be computed. The stress in the web is limited to f_{yw}, which in this case is 355 N/mm^2. Knowing the stresses and cross sectional dimensions, the tension force and compression force can be calculated, compared, and the position of the neutral axis adjusted until equilibrium is achieved. This is a job best left to electrons within a spreadsheet…
In this case, the solution is shown in Figure 6. Summing the product of the stress and area, the following forces are obtained:

Compression flange	440 × 400 × 20	= 3520000 N
web "plateau"	139 × 355 × 12	= 592140 N
web above "hole"	0.5 × (308+355) × 82 × 12	= 326196 N
web below "hole"	0.5 × 207 × 363 × 12	= 450846 N
	Summation	= 4890 kN
Tension flange	405 × 400 × 20	= 3240000 N
web "plateau"	355 × 77 × 12	= 328020 N
web	0.5 × 622 × 355 × 12	= 1324860 N
	Summation	= 4890 kN

Equilibrium of force has been achieved.

Moment resistance

Once equilibrium has been found, the moment resistance is simply the summation of the force in each element, multiplied by the lever arm.
3520000 × 771 = 2.71 × 10^9 Nmm
592140 × 692 = 409 × 10^6 Nmm

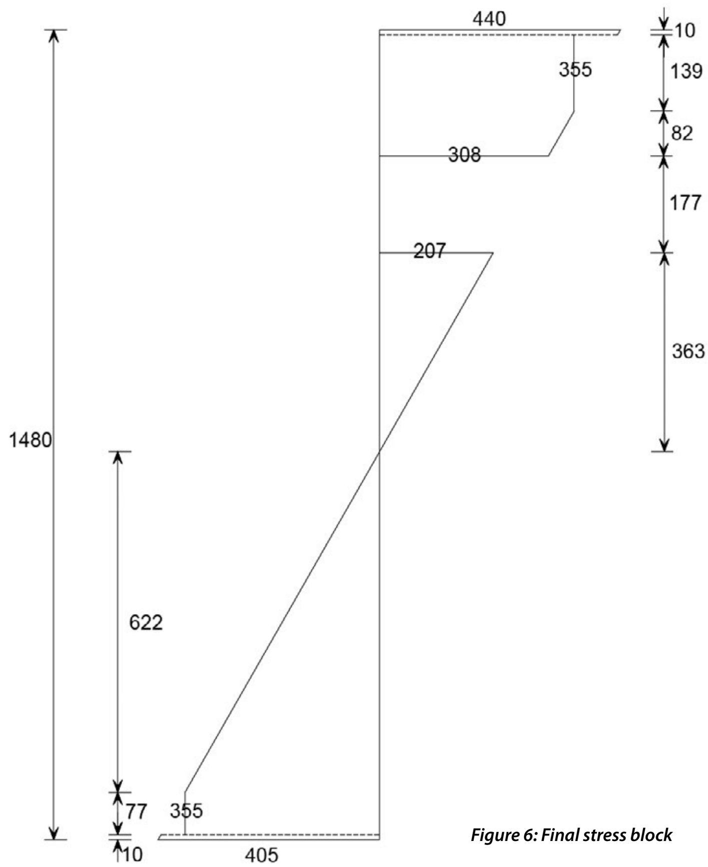

Figure 6: Final stress block

326196 × 581 = 189.5 × 10^6 Nmm
450846 × 2/3 × 363 = 109 × 10^6 Nmm
3240000 × 709 = 2.30 × 10^9 Nmm
328020 × 661 = 217 × 10^6 Nmm
1324860 × 2/3 × 622 = 549 × 10^6 Nmm
Moment resistance = 6485 kNm

Conclusions to Part 1

Despite how it may appear at first glance, this process is not overly onerous and is suited to a spreadsheet application – perhaps with some VBA to determine the neutral axis. Different solutions can then be readily examined and resistance calculated. In Part 2, the lateral torsional buckling resistance and the shear resistance will be calculated.