The aim of this feature is to share up-dates, design tips and answers to queries. The Steel Construction Institute provides items which, it is hoped, will prove useful to the industry.

AD 254

Design Considerations for the Vibration of Floors – Part 2

Introduction

AD 253 *Design considerations for the vibration of floors* warned that where vibrations disturb the occupants of a building it is usually excessive accelerations that cause the disturbance. It also warned that having the frequency of the floor above a certain threshold frequency, such as 3Hz, does not necessarily guarantee that the floor will prove satisfactory in service. In addition, AD 253 described the forcing function produced by walking activities and summarised the dynamic behaviour of floor structures. This Advisory Desk article follows on from AD 253 by giving information on acceptable levels of acceleration and explaining how the expected acceleration may be calculated.

Acceptability criteria

The evaluation of the exposure of humans to vibrations within buildings is covered by BS 6472: 1992 *Guide to evaluation of human exposure to vibration in buildings (1Hz to 80Hz)*, which is written to cover many vibration environments in buildings. This Standard presents acceleration limits for vibrations as a function of the exposure time and the frequency. It contains graphs of acceleration against frequency for both lateral and vertical vibration. These graphs are called "base curves". The "base curve" for vertical vibration is in Figure 1. Note that the acceleration is given as the rms (root mean square) value.

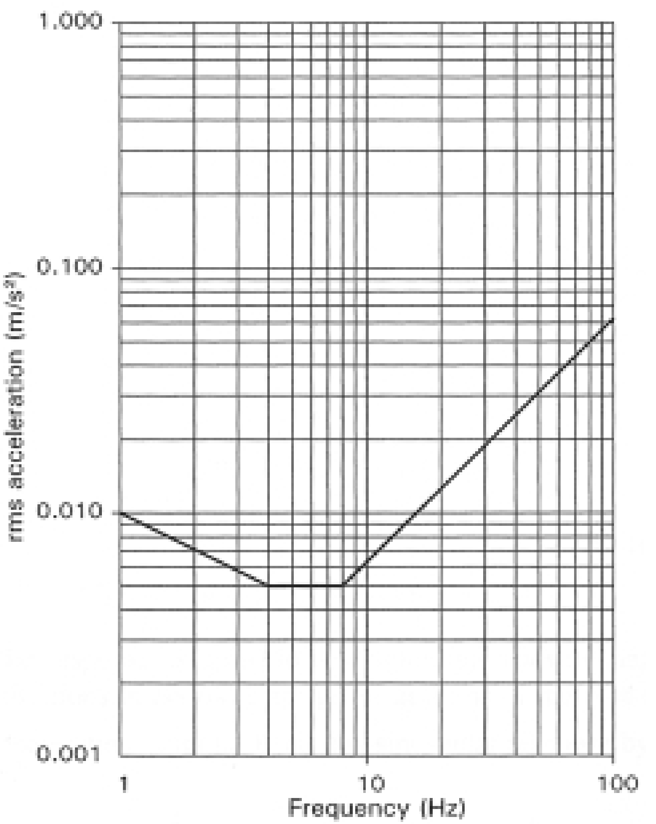

Fig. 1. Building vibration z-axis base curve for root-mean-square (rms) acceleration according to BS 6472: 1992.

The accelerations acceptable for different uses of buildings are described using the "base curve". Multiplying factors are used to increase the acceleration level of the "base curve" according to the intended use of the building. These multiplying factors are referred to as 'response factors' within the SCI guide P 076, *Design Guide on the Vibration of Floors*. Suitable design values for different office accommodation will also be given in a future Advisory Desk article.

The maximum calculated value of acceleration is found as follows. The "base curve" acceleration, a_{base}, is found from Figure 1 for the fundamental frequency of the floor. This acceleration, a_{base}, is multiplied by the response factor, R, appropriate to the use of the floor, to give a limiting acceleration $a_{base} \times R$.

For example, for a floor with the fundamental frequency equal to 5Hz, the "base curve" acceleration is 0.005 m/sec^2 rms. If the R value appropriate for the use of the building is 8, then acceptable acceleration is $0.005 \times 8 = 0.04$ m/sec^2 rms.

Calculation of the accelerations

Resonance can occur even if the fundamental frequency of the floor is above a given minimum design value. This is because vibrations arise from components of the walking activity. These components of walking activity occur because the force versus time graph of walking is made up of many different sine curves, as shown in AD 253. The largest response will generally occur when the lowest whole number multiple (*harmonic*) of the activity frequency is equal to the fundamental frequency of the floor (i.e., resonance). For example, consider a floor with a fundamental frequency f_0 = 4.6 Hz. The frequency range for walking activities is 1.7 Hz to 2.4 Hz, so the pace rate that would cause the highest floor response would be from the *second harmonic* of the walking activity (2×2.3 Hz = 4.6 Hz). If the floor design was changed so that the fundamental frequency was raised to 6.0Hz, the highest floor response would be from the *third harmonic* of the walking activity (3×2.0 Hz = 6.0 Hz).

The peak acceleration response is calculated from the following equation:

$$a_{peak} = \frac{P_0 \alpha_n}{M} \frac{1}{2\zeta} \qquad (1)$$

where P_0 is the weight (=mass × gravity) of the person in Newtons (N),
α_n is the Fourier coefficient of the nth harmonic component of the walking activity,
M is the effective vibrating mass (kg)
and ζ is the damping ratio.

(The above equation can be derived from AD 253 from the equation for a_{base}, using $D_{\beta=1}$ as the value of the Dynamic magnification factor.)

The response assumed in developing the above equation is purely sinusoidal, whereas the limits to vibrations in BS 6472 are given in terms of the root-mean-square (rms) acceleration a_{rms}. The rms acceleration is found from this sinusoidal response by dividing a_{base} by $\sqrt{2}$.

The value of $a_{peak}/\sqrt{2}$ should be compared with acceleration from the curve in Figure 1, a_{base}, (for the fundamental frequency of the floor) multiplied by the response factor, R, appropriate to the use of the floor. If $a_{peak}/\sqrt{2}$ is greater than $a_{base} \times R$, the floor may be unsuitable for the intended use.

The mass of a person is commonly taken as 76 kg.

The appropriate Fourier coefficient of the walking activity, α_n, is the value that coincides with the fundamental frequency of the floor, which may be calculated by the methods given below. Simple recommendations for design values of α_n will be given in a future Advisory Desk article. The highest values of α_n occur at the first harmonic of walking activity, as shown in AD 253 Fig. 3. Therefore, minimum frequency limits are normally given in design guides and codes of practice to ensure that the fundamental frequency of the structure does not coincide with the first harmonic of walking activities. By ensuring that the fundamental frequency of the floor is above the frequency range for the first harmonic component of walking activity, the magnitude of the floor acceleration is reduced because the higher harmonic components of the walking activity have lower values of α_n. However, as can be seen from Equation (1), the acceleration response from the floor may still be very high if the mass is too low. A practical example of a floor with a low mass would be a steel framed structure with a timber floor.

The response calculation in Equation (1) uses only one harmonic of the walking activity, which is the harmonic that coincides with the fundamental frequency of the floor. This is normally sufficient for checking the response to walking activity. However, for more vigorous activities such as dancing, aerobics etc, it is recommended that more harmonics are considered.

Floor mass for vibration and the effective vibrating mass, *M*

For offices in the UK, 10% of the design imposed loading is normally considered as permanent, and represents the sensible loading on a furnished floor. In the Eurocodes, the proportion of the imposed loading considered is much higher, and corresponds to the permanent loads plus a "frequent variable action" combination factor, Ψ_1, times the imposed load. For offices, the UK National Application Document (NAD) for Eurocode 3 Part 1, DD ENV 1993-1-1, gives $\Psi_1 = 0.6$ which implies that a very large proportion of the imposed load is composed of office furniture and stored materials such as books and paper.

The effective vibrating mass M may be calculated using the SCI publication P 076. This takes M as the floor mass per unit area (comprising the self weight, and other permanent loads, plus a proportion of the imposed load) multiplied by one-quarter of the effective vibrating area. P 076 gives a method to calculate the effective length, L_{eff} of the effective vibrating area and the effective width, S, of the effective vibrating area. Therefore, M is calculated from:

$$M = \frac{mL_{eff}S}{4} \qquad (2)$$

where m is mass per unit area (kg/m^2) of the floor plus any loading that is considered to be permanent.

Fundamental frequency

When individual structural components are inter-connected to form a complete floor system and this floor system vibrates, the whole floor structure moves up and down in a particular form, known as a *mode shape*. Although, each floor frequency has a particular mode shape associated with it, it is generally the lowest (1st mode) or fundamental frequency that is of particular interest in design (see Figure 2), because the largest acceleration response is normally found when this mode is excited to resonance. The *fundamental frequency* of the floor is lower than the frequency of any of the individual structural components.

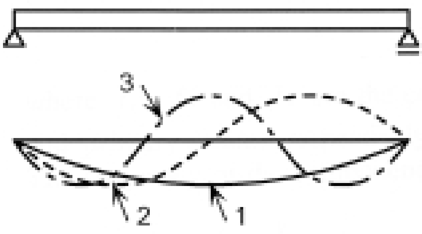

Fig. 2. Beam mode shapes (first three frequencies).

In conventional composite floor systems, the fundamental frequency may be estimated by using engineering judgement on the likely mode shape, considering how the supports and boundary conditions will affect the behaviour of the individual structural components. For example, on a simple floor comprising a slab which is continuous over a number of secondary beams and these secondary beams are supported by stiff primary beams, there are two possible mode shapes that may be sensibly considered:

1. Secondary (floor) beam mode
The primary beams form nodal lines (i.e. they have zero deflexion) about which, the secondary beams vibrate as simply-supported members (see Figure 3(a)). In this case, the slab flexibility is affected by the approximately equal deflexions of the supports. As a result of this, the slab frequency is assessed on the basis that fixed-ended boundary conditions exist.

2. Primary (main) beam mode
The primary beams vibrate about the columns as simply-supported members (see Figure 3(b)). Using a similar reasoning as above, due to the approximately equal deflexions at their supports, the secondary beams (as well as the slab) are assessed on the basis that fixed-ended boundary conditions exist.

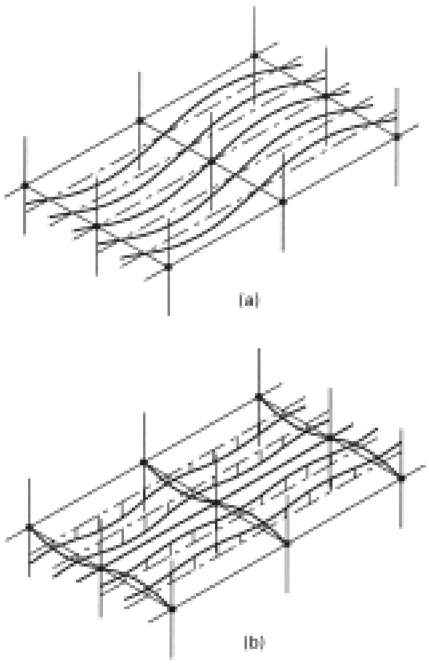

Fig. 3. Typical fundamental mode shapes for composite floor systems (a) governed by secondary beam flexibility (b) governed by primary beam flexibility.

The lowest frequency value determined by consideration of these two cases defines the fundamental frequency of the floor f_0 (and its corresponding mode shape). As steel-concrete composite construction is essentially an overlay of one-way spanning elements, the frequency of the whole floor system can be calculated for each mode shape, by summing the deflexion calculated from each of the above components, and placing this value within Equation (7). Alternatively, it can sometimes be convenient to use these component frequencies directly, to evaluate the fundamental frequency of the floor by Dunkerly's approximation shown in Equation (3) below. These two methods give identical results.

$$\frac{1}{f_0^2} = \frac{1}{f_1^2} + \frac{1}{f_2^2} + \frac{1}{f_3^2} \tag{3}$$

where f_1, f_2 and f_3 are the component frequencies (Hz) of the composite slab, secondary beams and primary beams respectively, with their appropriate boundary conditions, as defined above. The frequency of the components may be calculated using the methods given below.

Natural frequency of components

The natural frequency of a simple spring-mass system as shown in AD 253 Figure 1(a) is given by:

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{4}$$

where f is the natural frequency (Hz), T is the period for one complete cycle of movement (seconds), k is the stiffness, and m is the mass.

The mass used is the mass relevant to vibration calculations. It is normally taken in the UK as the mass from permanent loads plus 10% of the design imposed loading, but the Eurocodes may suggest adopting a higher load as explained above in *Floor mass for vibration and the effective vibrating mass, M*.

The natural frequency of a beam of uniform section is given by:

$$f = C_B\sqrt{\frac{EI}{mL^4}} \tag{5}$$

where EI is the flexural rigidity of the member (Nm^2), m is the mass per unit length (kg/m), L is the span of the member (m), and C_B is the frequency factor representing the beam support and/or loading conditions.

Some standard values of C_B for elements with different boundary conditions are as follows:

pinned/pinned ('simply-supported')	$\pi/2$
fixed/pinned (propped cantilever)	2.45
fixed both ends	3.57
fixed/free (cantilever)	0.56

For uniform beams which are continuous over supports (e.g., secondary beams connected either side of a primary beam web), the natural frequency will increase when one span is stiffer (shorter) than the main span. However, when the spans are equal, the natural frequency will be the same as for a simply-supported beam (i.e., $C_B = \pi/2$). A graphical representation of this effect on C_B for 2- and 3-span uniform beams is shown in Figure 4 below.

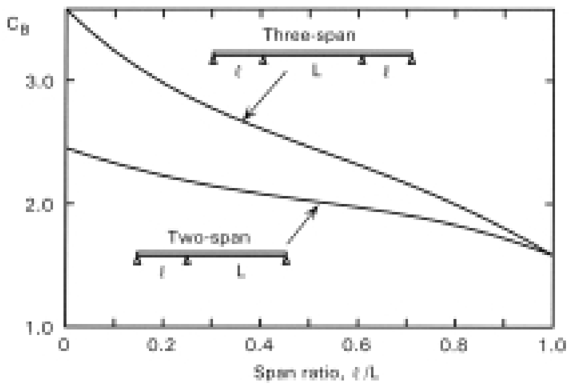

Fig. 4. Frequency factor C_B for uniform continuous beams.

The natural frequency, f, of a simply supported beam may be determined conveniently from the maximum deflexion δ when the beam is loaded with mass m appropriate for the calculation of frequency. For a simply-supported beam, with a uniformly distributed load, this is:

$$\delta = \frac{5wL^4}{384EI} = \frac{5mgL^4}{384EI} \tag{6}$$

where g is the acceleration due to gravity (i.e., 9.81 m/s^2).

Rearranging Equation (6), and substituting the value of m and $C_B = \pi/2$ into Equation (5), gives the well known natural frequency expression that is often used in design:

$$f = \frac{17.8}{\sqrt{\delta}} \approx \frac{18}{\sqrt{\delta}} \tag{7}$$

where δ is the maximum deflexion in millimetres due to the self weight and other permanent loads.

Note that pre-cambering will not affect the natural frequency of the beam because it does not affect the stiffness of the element.

The above gives a general over-view of the prediction of the acceleration and fundamental frequency of floors. These calculations are most commonly applied to steel-concrete composite floors, but the general principles apply to other types of construction, such as floors of pre-cast concrete or timber and floors in reinforced concrete provided the appropriate damping factors are used.

Contact: Dr Stephen Hicks, SCI. Tel: 01344 623345.
E-mail: s.hicks@steel-sci.com