Members with axial load and moments – at elevated temperatures

In what could be the final article on structural steel design at elevated temperatures (at least until Gen2 in 2028!) David Brown of the SCI looks at the general case of members subject to combined axial and bending.

A reader of *New Steel Construction* has noted that the design of members subject to bending and members subject to an axial force had been covered – but both effects in splendid isolation, and observed that the general case was to have combined moment and axial effects. Never one to shirk a challenge, this article is the result.

Overall plan

Designers will know that in normal design (at ambient temperatures) the resistance of members to combined axial load and bending is covered by expression 6.61 and 6.62 of BS EN 1993-1-1. Reference to that pair of expressions is usually sufficient to dampen any further enthusiasm – the expressions are painful to work through by hand.

At elevated temperatures, there are similar looking pairs of expressions in BS EN 1993 1-2. Expressions 4.21a and 4.21b cover Class 1 and 2 sections, and expressions 4.21c and 4.21d cover Class 3 sections. As might be expected the expressions for Class 3 are the same as those for Class 1 and 2, but utilising the elastic modulus in place of the plastic modulus. Class 4 sections are not covered in the same way at all – SCI advice is generally to choose a different section.

Within the expressions 4.21a and 4.21b, the ratios follow the familiar form of $\frac{\text{effect}}{\text{resistance}}$, both terms being affected by reductions in the fire limit state. The calculation of resistance in bending and in axial has been covered in previous articles. The final answer is the summation of the ratios for axial, major axis bending and minor axis bending.

In the same way that expressions 6.61 and 6.62 have interaction factors, expressions 4.21a and 4.21b have factors k_{LT}, k_y and k_z. The factors depend on the shape of the bending moment diagram and the utilisation in compression, so are in principle familiar to anyone who has looked in detail at the ambient design.

The expressions

Expressions 4.21a and 4.21b are reproduced below:

$$\frac{N_{fi,Ed}}{\chi_{min,fi}Ak_{y,\theta}\frac{f_y}{\gamma_{M,fi}}}+\frac{k_y M_{fi,Ed}}{W_{pl,y}k_{y,\theta}\frac{f_y}{\gamma_{M,fi}}}+\frac{k_z M_{z,fi,Ed}}{W_{pl,z}k_{y,\theta}\frac{f_y}{\gamma_{M,fi}}}\le 1 \qquad 4.21a$$

$$\frac{N_{fi,Ed}}{\chi_{z,fi}Ak_{y,\theta}\frac{f_y}{\gamma_{M,fi}}}+\frac{k_{LT} M_{y,fi,Ed}}{\chi_{LT,fi}W_{pl,y}k_{y,\theta}\frac{f_y}{\gamma_{M,fi}}}+\frac{k_z M_{z,fi,Ed}}{W_{pl,z}k_{y,\theta}\frac{f_y}{\gamma_{M,fi}}}\le 1 \qquad 4.21b$$

Within the first term of expression 4.21a, the minimum value of the reduction factor χ_{fi} will usually be $\chi_{z,fi}$ in the minor axis. It would be unusual to have the minimum slenderness in the major axis.

Considering the second term, it seems almost certain that 4.21b will be critical, since the reduction factor $\chi_{LT,fi}$ appears in the denominator and is always 1.0 or (significantly) less. It may be possible that this ratio in 4.21a is critical, but only at very short lengths – recognising the values of k_y and k_{LT} have not been considered yet and could potentially change the conclusion.

The third term is the same in both expressions, so a casual review suggests that 4.21b is the likely candidate to be critical.

A numerical example

It seems that some readers value a numerical example, if only to check their own spreadsheet calculations. This example considers a 203 UC 60 in S355, 4m long, at 500°C. The bending moments in the fire condition are at one end of the column, diminishing to zero at the other end. The shape of the bending moment diagram means that C_1 = 1.77.

Classification

The first step is to classify the section to determine which pair of expression should be verified. Member class may change at elevated temperature, because the value of ε is modified.

At elevated temperature, $\varepsilon = 0.85\sqrt{\frac{235}{f_y}} = 0.85 \times \sqrt{\frac{235}{355}} = 0.69$

The Class 2 limit for the flange is $10\varepsilon = 10 \times 0.69 = 6.9$

The actual $\frac{c_f}{t_f} = 6.2$, so the flange is at least Class 2.

The classification of the web in combined axial load and bending requires the axial load, which is 650 kN in the fire limit state.

From the expression in P362,

$$\alpha = ½\left(1+\frac{N_{Ed}}{f_y c t_w}\right) = ½\left(1+\frac{650\times 10^3}{355\times 160.8\times 94}\right) = 1.106$$

therefore α takes the limiting value of 1.0.

As $\alpha > 0.5$ then the Class 2 limit is given by:

$$\frac{456\varepsilon}{13\alpha - 1} = \frac{456\times 0.69}{13\times 1.0 - 1} = 26.2$$

The actual $\frac{c_w}{t_w}$ =17.1 so the web and the whole section is at least Class 2.

For Class 2 sections, the pair of expressions 4.21a and 4.21b must be verified.

Design data

The design resistances at ambient and at 500°C are shown below, calculated as shown in previous articles, with the design effects in the fire limit state.

Ambient	$N_{b,z,Rd}$ = 1450 kN	$M_{b,Rd}$ = 233 kN	
At 500°C	$N_{b,z,fi,Rd}$ = 893 kN	$M_{b,fi,Rd}$ = 123 kN	
Effects	$N_{fi,Ed}$ = 650 kN	$M_{y,fi,Ed}$ = 40 kN	$M_{z,fi,Ed}$ = 10 kN

The calculation of $N_{b,z,fi,Rd}$ does not apply any reduction to the buckling length that would be permitted in Figure 4.1 of BS EN 1993-1-2. The value of 650 kN selected for $N_{fi,Ed}$ is relatively low. If the column was utilised to around 85% at ambient, a reasonable value for $N_{fi,Ed}$ might be 0.85 × 1450 × 0.65 = 801 kN. The reduction of 0.65 is based on the simplification given in Note 2 to Figure 2.1 of BS EN 1993-1-2. Previous articles have noted that this reduction is conservative, so the value of 650 kN can be seen as a more realistic value.

Intermediate values in compression

At 500°C, the value of $k_{y,\theta}$ is 0.78 and the value of $k_{E,\theta}$ is 0.60.

Following the approach recommended in previous articles, the salient calculation values are shown below, at a temperature of 500°C. Note that the values in the major axis will be required later in the calculations. As the steel is S355, α = 0.529.

	Minor axis	Major axis
$\bar{\lambda}_{fi}$	1.148	0.667
χ_{fi}	0.422	0.666
$N_{b,fi,Rd}$	893 kN	1410 kN

▶ 40

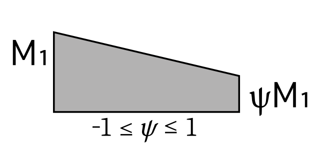
▶ 38

Intermediate values in bending at 500°C

	Minor axis
$\bar{\lambda}_{LT,fi}$	0.644
$\chi_{LT,fi}$	0.678
$M_{b,fi,Rd}$	123.3 kN

Interaction coefficients

The values of coefficients k_y, k_z and k_{LT} involve the shape of the bending moment diagram and an equivalent uniform moment factor β_M. In very many cases, the bending moment diagram in the column will be linear (with no significant loading applied along the length of the column). In the case of a linear bending moment diagram, the value of β_M is shown below.



$\beta_{M,\psi} = 1.8 - 0.7\psi$

In this instance, since in both axes $\psi = 0$ then $\beta_{M,\psi} = 1.8$

The coefficients are straightforward to calculate, but designers should pay attention to the latest version of the code. In earlier versions of the code, the expressions for μ_y and μ_z were unfortunately reversed and modified.

Then $k_{LT} = 1 - \dfrac{\mu_{LT} N_{fi,Ed}}{\chi_{z,fi} A k_{y,\theta} \dfrac{f_y}{\gamma_{M,fi}}}$ with $\mu_{LT} = 0.15\bar{\lambda}_{z,\theta}\,\beta_{M,LT} - 0.15 \le 0.9$

$\mu_{LT} = 0.15 \times 1.148 \times 1.8 - 0.15 = 0.16$

In the expressions for the interaction coefficients it is disappointing that the denominator is not simply shown as the relevant resistance at elevated temperature.

$$k_{LT} = 1 - \frac{0.16 \times 650}{893} = 0.88$$

The second coefficient is k_y, given by:

$$k_y = 1 - \frac{\mu_y N_{fi,Ed}}{\chi_{y,fi} A k_{y,\theta} \dfrac{f_y}{\gamma_{M,fi}}} \le 3$$

with $\mu_y = (2\beta_{M,z} - 5)\,\bar{\lambda}_{y,\theta} + 0.44\beta_{M,y} + 0.29 \le 0.8$ and $\lambda_{y,20°C} \le 1.1$

In this case, the non-dimensional slenderness was 0.585 at ambient temperature, so the limiting value of 1.1 does not apply.

$\mu_y = (2 \times 1.8 - 5) \times 0.667 + 0.44 \times 1.8 + 0.29 = 0.149$

Note that the non-dimensional slenderness in the major axis is required.

$$k_y = 1 - \frac{0.149 \times 650}{1410} = 0.931$$

The final coefficient is k_z, given by:

$$k_z = 1 - \frac{\mu_z N_{fi,Ed}}{\chi_{z,fi} A k_{y,\theta} \frac{f_y}{\gamma_{M,fi}}} \leq 3$$

with $\mu_z = (1.2\beta_{M,z} - 3)\bar{\lambda}_{z,\theta} + 0.71\beta_{M,y} - 0.29 \leq 0.8$

$\mu_z = (1.2 \times 1.8 - 3) \times 1.148 + 0.71 \times 1.8 - 0.29 = 0.024$

$$k_z = 1 - \frac{0.024 \times 650}{893} = 0.983$$

Bringing it all together

Substituting all the information into expressions 4.21a and 4.21b, the results are:

$$\frac{650}{893} + \frac{0.931 \times 40 \times 10^6}{656 \times 10^3 \times 0.78 \times 355} + \frac{0.983 \times 10 \times 10^6}{305 \times 10^3 \times 0.78 \times 355} = 1.049$$

and

$$\frac{650}{893} + \frac{0.88 \times 40 \times 10^6}{0.678 \times 656 \times 10^3 \times 0.78 \times 355} + \frac{0.983 \times 10 \times 10^6}{305 \times 10^3 \times 0.78 \times 355} = 1.131$$

In this case, at the end of the process, the column is unsatisfactory. In practice, designers will not start the process with a temperature, but will know the selected column and the effects in the fire limit state - and need to calculate the critical temperature. This is easy if the calculations are embedded in a spreadsheet. The critical temperature is found to be 441°C in this instance. The specification for the necessary protection is therefore to limit the temperature of this steel member to no more than 441°C at the required period of fire resistance.

Conclusions

The primary purpose of this article is to help those designers wishing to correctly determine a critical temperature, especially those preparing their own spreadsheet solution. The process is very similar to the verification at ambient temperature with a pair of interaction expressions to satisfy after determining intermediate values. After inspecting the expressions in BS EN 1993-1-2, it does seem likely that expression 4.21b will be critical.

This example also serves as a reminder that the critical temperatures in Table NA.1 of the UK NA to BS EN 1993-1-2 are limited to the case of members in pure compression – and should not be used if any moment is introduced to the section. ■