Design at elevated temperatures – unrestrained beams

Previous articles on the design of members at elevated temperatures[1,2], covered restrained beams and columns. This third article in the series by David Brown of the SCI covers the verification of unrestrained beams.

Introduction

Unrestrained beams are rather like columns – the simple reduction in design strength which was satisfactory for restrained members is not appropriate. For both unrestrained beams and unrestrained columns, buckling behaviour – which is non-linear even at ambient temperatures – is impacted by changes to the yield strength and changes to the modulus of elasticity. The overall process is to firstly calculate the reduced loading in the fire limit state, which was covered previously. The LTB resistance can be calculated at any given temperature, which must of course be greater than the design effects (the bending moment in the fire limit state). The critical temperature is when the design resistance just exceeds the design effects. This critical temperature together with the A_m/V ratio can be used by a fire protection company to specify the necessary thickness of their product.

Changes to the resistance calculation

The changes specified in BS EN 1993-1-2 are straightforward.

A revised non-dimensions slenderness $\bar{\lambda}_{LT,\theta,com}$ is required, given by

$$\bar{\lambda}_{LT,\theta,com} = \bar{\lambda}_{LT,}[k_{y,\theta,com}/k_{E,\theta,com}]^{0.5}$$

The values of $k_{y,\theta,com}$ and $k_{E,\theta,com}$ adjust the material strength and modulus of elasticity (Young's Modulus) respectively and are taken from Table 3.1 of BS EN 1993-1-2.

Although many designers will know that the strength of steel does not reduce until after 400°C, the modulus of elasticity is modified as soon as the temperature reaches 200°C. This means that the adjustment factor $[k_{y,\theta,com}/k_{E,\theta,com}]^{0.5}$ is significant at relatively low temperatures. The relationship between the adjustment factor and temperature is shown in Figure 1. Since the adjustment factor is greater than 1.0, the slenderness is increased, leading to a more significant reduction factor.

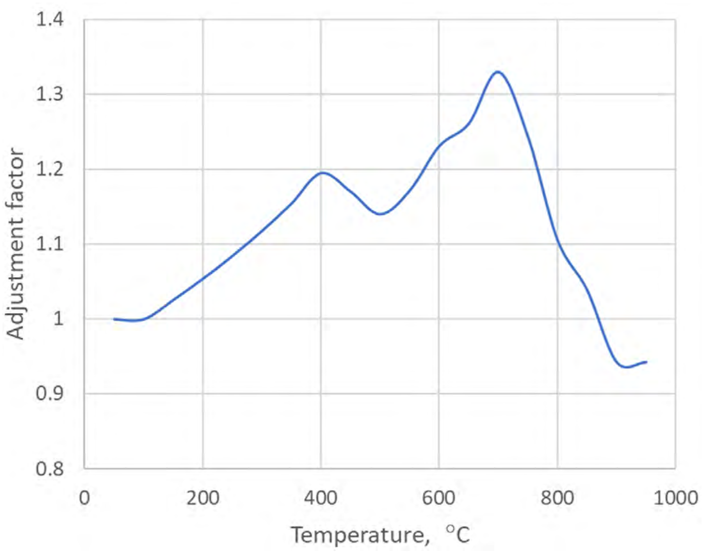

Figure 1: Variation of $[k_{y,\theta,com}/k_{E,\theta,com}]^{0.5}$ with temperature

The second change is that the imperfection factor α is no longer related to the cross section geometry, but is a fixed value given by:

$$\alpha = 0.65\sqrt{235/f_y}$$

The final changes are subtle alterations to the formulae to calculate the reduction factor $\chi_{LT,fi}$. In particular, there is no plateau length within the expressions, so the reduction applies even at low values of slenderness.

The final resistance is given by:

$$M_{b,fi,t,Rd} = \chi_{LT,fi} W_{pl,y} k_{y,\theta,com} f_y / \gamma_{M,fi}$$

(for Class 1 and Class 2 sections, hence the use of W_{pl})

Due to the combined impact of the changes in the formulae, even at 20°C there is a marked reduction in the calculated LTB resistance. Table 1 shows the difference, for a 533 × 210 × 82 UB in S355 with a C_1 value of 1.0. In a perfect world one would have hoped the resistance at 20°C was the same as the "cold" value, but the differences in the formulae preclude this.

Table 1: Comparison of LTB resistance

Beam buckling length (m)	"Cold" Resistance (kNm)	"Hot" resistance (kNm) – at only 20°C
9	193	147
6	312	238
3	577	451

Design example

Verification at ambient temperature:

7m span beam, unrestrained, nominally pinned supports, subject to a UDL. The variable action is from office loading.

G_k = 3.0 kN/m²; Q_k = 3.3 kN/m². Beams spaced at 3.6 m centres.

Using expression 6.10 from BS EN 1990, the design combination of actions is:

1.35 × 3.0 + 1.5 × 3.3 = 9.0 kN/m²

The design load on the beam = 9.0 × 3.6 = 32.4 kN/m and the maximum bending moment $= \frac{32.4 \times 7^2}{8} = 199$ kNm

Looking in the Blue Book, a 406 × 178 × 74 UB in S355 appears appropriate.

$M_{b,Rd}$ = 218 kNm, > 199 kNm, OK.

Deflection is unlikely to be a critical check with unrestrained beams, but for completeness is verified.

Characteristic variable load on beam = 3.3 × 3.6 = 11.9 kN/m

$$\delta = \frac{5 \times 11.9 \times 7000^4}{384 \times 210000 \times 27300 \times 10^4} = 6.5 \text{ mm}$$

$$\text{Allowable} = \frac{7000}{360} = 19.4, \text{ OK}$$

In preparation for the verification at elevated temperature, the non-dimensional slenderness $\bar{\lambda}_{LT}$ is required. The non-dimensional slenderness requires M_{cr}, which must be calculated using software or from a formula (for example as given in P362).

In this instance, the formula has been used, and M_{cr} = 253 kNm

Then $\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} = \sqrt{\frac{1500 \times 10^3 \times 355}{253 \times 10^6}} = 1.45$

(noting that the section is Class 1, so $W_y = W_{pl}$ and the flange is 16.0 mm, so f_y = 355 N/mm²)

▸26

▸24 *Verification at elevated temperature*

Firstly, the value of the actions in the fire limit state is calculated using the expressions in BS EN 1993-1-2.

The reduction factor to be applied to the design loads, η_{fi}, is given by:

$$\eta_{\mathrm{fi}} = \frac{G_k + \psi_{\mathrm{fi}} Q_k}{\gamma_G G_k + \gamma_Q Q_k} = \frac{3 + 0.5 \times 3.3}{1.35 \times 3 + 1.5 \times 3.3} = 0.52$$

The design effects in fire $E_{\mathrm{d,fi}}$ are therefore $E_{\mathrm{d,fi}} = \eta_{\mathrm{FI}} E_{\mathrm{d}} = 0.52 \times 199 =$ 104 kNm.

The calculation steps follow those outlined above - calculate a revised value of α (for $f_y =$ 355, $\alpha = 0.53$) and then for each temperature:

- Calculate a revised value of the non-dimensional slenderness, $\bar{\lambda}_{\mathrm{LT},\theta,\mathrm{com}}$
- Calculate a new reduction factor $\chi_{\mathrm{LT,fi}}$
- Calculate a new buckling resistance $M_{\mathrm{b,fi,t,Rd}}$ – remembering that the factor $k_{y,\theta,\mathrm{com}}$ appears in the final calculation as well as the revised slenderness calculation.

A spreadsheet will facilitate these calculations. For the selected beam, the relationship between the moment resistance $M_{\mathrm{b,fi,t,Rd}}$ and temperature is shown in Figure 2.

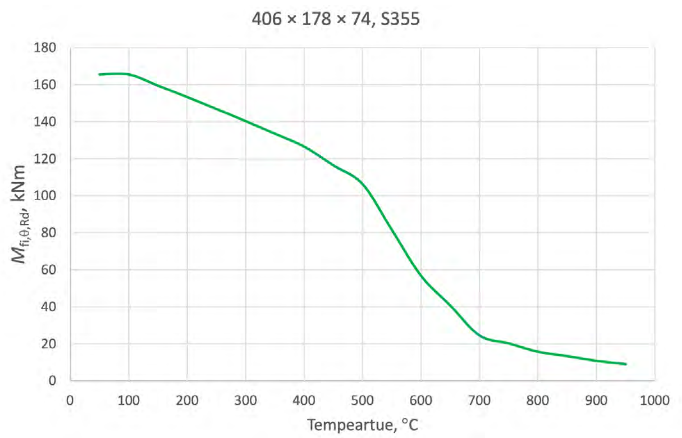

Figure 2: LTB resistance at elevated temperature

From Figure 2, it can be seen that the design resistance falls below 104 kNm at around 500°C. The precise figure is 505°C, which is used in the following calculations.

Interpolating from Table 3.1 of BS EN 1993-1-2:

$k_{y,\theta} = 0.765$

$k_{y,\theta} = 0.586$

Then $\bar{\lambda}_{LT,\theta,com} = \bar{\lambda}_{LT,}[k_{y,\theta,com}/k_{E,\theta,com}]^{0.5} = 1.45 \times \left(\frac{0.765}{0.586}\right)^2 = 1.66$

With $\alpha = 0.53$

$\phi_{LT,\theta,com} = 0.5 \times [1 + 0.53 \times 1.66 + 1.66^2] = 2.32$

and

$$\chi_{LT,fi} = \frac{1}{2.32\sqrt{2.32^2 - 1.66^2}} = 0.25$$

then

$M_{fi,t,Rd} = (0.25 \times 1500 \times 10^3 \times 0.765 \times 355)/(1.0\times10^6) = 102$ kNm

(or, with more precision in the intermediate values, 103.7 kNm)

Shear resistance

Just as deflection is unlikely to be critical with unrestrained beams, so is shear. The shear resistance at elevated temperatures is given by:

$V_{fi,t,Rd} = k_{y,\theta,web}V_{Rd}\,[\gamma_{M0}/\gamma_{M,fi}]$

Which uses the same value of $k_{y,\theta}$ calculated previously.

From the Blue Book, $V_{Rd} = 858$ kN

$V_{fi,t,Rd} = 0.765 \times 858[1/1] = 656$ kN

The design shear load in the fire condition is 0.52 × 32.4 × 7/2 = 59 kN, OK.

Conclusions

In this particular example, the critical temperature was 505°C. The unrestrained condition is intuitively more onerous than when the beam is restrained, so the tabulated critical temperatures for restrained beams (discussed in part 1) should not be used. In the fire condition, the example beam is "utilised" at 104/166 = 0.63. The resistance of 166 kNm is the LTB resistance at 20°C. Incorrectly using Table NA.1 of the UK NA to BS EN 1993-1-2, the critical temperature might be assessed as around 548°C, which is not adequate.

Table 18 of the ASFP Yellow Book includes "beams not carrying concrete floor slabs" and presents a limiting temperature of 585°C for offices, which is similarly not adequate. The tabulated values in these two documents, which are for restrained beams, should not be used for unrestrained beams.

The correct calculation process for unrestrained beams is not difficult and is readily facilitated in a spreadsheet. ■

1. Fire protection of steelwork, NSC, March 2024
2. Critical temperatures for fire design: Part 2 – Columns, NSC, April 2024