AD 471: **Second order moments for splice design**

AD notes 243 and 244 were produced when BS 5950:1990 was current. The present AD note provides an update for designers using BS EN 1993-1-1. The same principles are followed as those set out in the approach in BS 5950. The general advice on the design of splices in AD 243 is still relevant.

Splices in compression members designed in accordance with BS EN 1993-1-8 sub-para. 6.2.7.1(15) that are not close to points of restraint are subject to second order effects that must be taken into account. AD 470 identified these effects as the strut moment and, if design bending moments are present in the member, such moments are amplified by the axial force.

The complete list of internal second order moments that may be present in a splice connection is as follows:

1) The strut moments $M_{y,FB}$ and $M_{z,FB}$ due to the initial bow in the compression member amplified by the axial force (FB denotes flexural buckling);
2) A minor-axis moment $M_{z,LTB}$ produced by resisting lateral-torsional buckling due to major axis moments (LTB denotes lateral torsional buckling);
3) An additional major-axis moment $M_{y,Amp}$ due to applied major-axis moments amplified by the axial force;
4) An additional minor-axis moment $M_{z,Amp}$ due to applied minor-axis moments amplified by the axial force.

The second order moments listed above are assumed to vary as the sine over the length between points of inflexion in the buckled shape (the effective length). The maximum second order moment (at the mid-span of an unrestrained segment) is calculated and adjusted to the appropriate value at the splice location, positioned at a distance x along the length L between points of inflexion.

The external design forces and moments are N_{Ed}, $M_{y,Ed}$ and $M_{z,Ed}$ (axial force, major-axis bending and minor-axis bending, respectively) where the bending moments are the applied moments at the splice position. The bending moments used to determine the additional second order moments (items 3 and 4 above) are the maximum values of the applied moments about the relevant axes in the unrestrained buckling segment being considered (where the splice is located). The shape of the bending moment diagrams between points of inflexion can be accounted for by considering equivalent uniform moment factors.

Columns subjected to axial compression:
The second order effects described in Item 1 above need to be considered. The strut moments $M_{y,FB}$ and $M_{z,FB}$ should be considered about each axis but only about one axis at a time (the column cannot buckle about both axes at the same time). The second order bending moments due to strut action at the mid-span between points of inflexion can be calculated as follows:

$$M_{i,FB,max} = N_{Ed} \cdot e_{P\delta,i} \cdot \gamma_{M1}$$

where:

$e_{P\delta,i}$ is the bow imperfection accounting for the second-order effects equal to:

$$e_{P\delta,i} = e_{0,i} \cdot k_{amp,i} \text{ but } e_{P\delta,i} \geq L/200 \text{ if } N_{Ed} \geq 0.90 \, N_{b,Rd,i}$$

$e_{0,i}$ is the initial bow imperfection about axis 'i' equal to:

$$e_{0,i} = \frac{W_{el,i}}{A} \cdot \alpha \cdot (\bar{\lambda}_i - 0.20)$$

$k_{amp,i}$ is the amplification factor equal to:

$$k_{amp,i} = \frac{N_{cr,i}}{N_{cr,i} - N_{Ed}}$$

$N_{b,Rd,i}$ is the design flexural buckling resistance of the column according to EN 1993-1-1 section 6.3.1 about axis 'i';

γ_{M1} is the material partial factor for buckling resistance given by the relevant EN 1993 part;

$W_{el,i}$ is the elastic modulus of the cross-section about axis 'i';

A is the cross-section area;

α is the equivalent imperfection factor according to EN 1993-1-1 section 6.3.1.2 related to flexural buckling about axis 'i';

$\bar{\lambda}_i$ is the non-dimensional slenderness according to EN 1993-1-1 section

6.3.1.2 related to flexural buckling about axis 'i'

$N_{cr,i}$ is the elastic critical buckling load for flexural buckling (Euler load) about axis 'i'.

Note that the amplification factors $k_{amp,i}$ are equivalent to the Eurocode presentation in equation 5.4. For an individual element, $\alpha_{cr} = N_{cr,i} / N_{Ed}$, meaning that:

$$\frac{1}{1 - \frac{1}{\alpha_{cr}}} = \frac{1}{1 - \frac{1}{N_{cr,i}/N_{Ed}}} = \frac{N_{cr,i}}{N_{cr,i} - N_{Ed}}$$

Whilst the model described above can suitably address the phenomenon in most cases, studies have shown that for columns with low slenderness loaded close to their buckling capacity ($N_{Ed} \geq 0.90 \, N_{b,Rd}$), the second order effects could be underestimated. The recommended minimum value of $L/200$ for the bow imperfection accounting for the second-order effects ($e_{P\delta,i}$) ensures that the model gives safe results for such cases.

The strut moment at the splice position between points of inflexion ($M_{i,FB,sp}$) is given by:

$$M_{i,FB,sp} = \sin\left(\pi \cdot \frac{x}{L}\right) \cdot M_{i,FB}$$

If second order effects in frames have been accounted for using equivalent members (i.e. increasing the buckling lengths of columns) in accordance with 5.2.2(c), the moment at the splice location should be taken as $M_{i,FB}$.

The combined design actions to design the splice are:

1: $N_{Ed} + M_{y,FB,sp}$ or 2: $N_{Ed} + M_{z,FB,sp}$

Unrestrained beams subjected to bending moments:
The second order effects described in Item 2 above need to be considered for elements subjected to major axis moment. The second order moment due to lateral-torsional buckling at the mid-span between points of inflexion can be calculated as follows:

$$M_{z,LTB,max} = \left(\frac{1}{\chi_{LT}} - 1\right) \cdot \frac{W_{el,z}}{W_{el,y}} \cdot M_{y,Ed,max} \cdot \gamma_{M1}$$

but $M_{z,LTB,max} \geq \dfrac{6 \cdot E \cdot I_z}{125 \cdot L}$ if $M_{y,Ed,max} \geq 0.90 \, M_{b,Rd}$

where:

$W_{el,z}$ is the minor axis elastic modulus;

$W_{el,y}$ is the major axis elastic modulus;

$M_{y,Ed,max}$ is the maximum major-axis design bending moment along the length L between points of inflexion;

χ_{LT} is the buckling reduction factor for the relevant buckling curve according to EN 1993-1-1 section 6.3.2;

$M_{b,Rd}$ is the design lateral torsional buckling resistance according to EN 1993-1-1 section 6.3.2;

I_z is the second moment of area about the minor-axis z.

For cases where section EN 1993-1-1 sub-para. 6.3.2.3 (2) is applied, $\chi_{LT,mod}$ may be used to evaluate $M_{z,LTB,max}$.

Studies have also shown that beams with low slenderness loaded close to their buckling capacity ($M_{y,Ed,max} \geq 0.90 \, M_{b,Rd}$), the second order effects could be underestimated. The recommended minimum second order bending moment allows for a minimum out-of-plane bow at failure of $L/200$, conveniently expressed as a minimum bending moment.

The second order moment at the splice position between points of inflexion is given by:

$$M_{z,FB,sp} = \sin\left(\pi \cdot \frac{x}{L}\right) \cdot M_{z,LTB,max}$$

The assessment of the location of the points of inflexion may not be apparent to the designer and difficult to determine for some bending moment diagram shapes. Therefore, the definition of the the second-order bending moment at the splice location (from a distance x along the length L between points of inflexion) can be challenging. As a simplification, designers may wish to assume $M_{z,LTB,sp} = M_{z,LTB,max}$. For segments with a reasonably uniform bending moment or for simply supported beams with uniformly distributed loads, the problem is simplified and the equation above can be used.

The combined design actions to design the splice are:

$$M_{y,Ed} + M_{z,Ed} + M_{z,LTB,sp}$$

Beam-columns subjected to axial compression and bending moments:

The second order effects described in Items 1 to 4 above need to be considered for elements subjected to axial compression and bending. The additional second order moments due to amplification of the applied moment about the axis 'i' by the axial force are given by:

$$M_{i,Amp,max} = (k_{amp,i} - 1) \cdot C_{mi} \cdot M_{i,Ed,max}$$

where:

$M_{i,Ed,max}$ is the maximum design bending moment along the length L between points of inflexion (about axis 'i');

C_{mi} is the equivalent uniform moment factor, given by EN 1993-1-1 Table B.3 about axis 'i' (C_{my} or C_{mz}).

The second order moment at the splice position between points of inflexion ($M_{i,Amp,sp}$) is given by:

$$M_{i,Amp,sp} = \sin\left(\pi \cdot \frac{x}{L}\right) \cdot M_{i,Amp,max}$$

The combined design actions at the splice position are:

1: $N_{Ed} + M_{y,Ed} + M_{z,Ed} + M_{yFB,sp} + M_{y,Amp,sp} + M_{z,Amp,sp} + M_{z,LTB,sp}$

1: $N_{Ed} + M_{y,Ed} + M_{z,Ed} + M_{zFB,sp} + M_{y,Amp,sp} + M_{z,Amp,sp} + M_{z,LTB,sp}$

The splice should be verified for both combinations.

If $\dfrac{N_{Ed}}{N_{b,Rd,i}} + \dfrac{M_{Ed}}{M_{b,Rd}} > 0.9$ then:

$M_{i,FB,sp}$ should be calculated as if $N_{Ed} > 0.9 N_{b,Rd,i}$ as given above for columns, and $M_{z,LTB,sp}$ should be calculated as if $M_{Ed} > 0.9\ M_{b,Rd}$ as given above for unrestrained beams.

Complementary information

Further detail and calculation examples are provided in References 1 and 2. Details of the studies completed to establish the minimum values of $e_{P\delta,i}$ and $M_{z,LTB,max}$ referred to will be described in an NSC paper to appear at a later date.

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References

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2   Pimentel R, *Design of beam-column splice connections according to Eurocode 3*, NSC, October 2020