Portal frames with flexible joints using Kleinlogel-type formulae

Single-span portal frames with flexible joints can be analysed using formulae similar to those produced by Kleinlogel. Concept designs or "hand" checks of detailed designs can be carried out using such methods. Richard Henderson of the SCI discusses the background.

Introduction

Adolf Kleinlogel produced a book[1] containing formulae setting out the internal bending moments and direct forces in rigid jointed frames subject to different load cases. A selection of these for single storey goal-post and pitched portal frames have been reproduced in the Steel Designer's Manual (SDM) for several editions. Their use has probably dwindled over the years due to the promotion of additional economy using plastic design and the ease of analysis using finite element (FE) software packages. One of the limitations on the usefulness of the formulae published in the SDM is that none are included for frame deflections. The formulae were produced using elastic analysis assuming bending deformations only and can be derived using the slope-deflection equations.

In 2020, two technical articles on the calculation of joint stiffness in steel design[2,3] were published in *New Steel Construction* magazine. In anticipation of increasing implementation of joint flexibility in frame design, Kleinlogel's formulae have been developed to include joint stiffnesses.

Structural analysis and joint stiffness

The traditional UK approach to structural analysis of frames assumes that joints between members are either perfectly pinned or fully rigid. Members are also assumed to have negligible size compared with the frame geometry. In rigid-jointed frames, the distribution of bending moments and their deflections depends on the flexural stiffness of the elements which are assumed to be axially rigid. In general, if a joint is designed to resist the calculated bending moment, the effect of joint stiffness is assumed to be negligible and this assumption produces reasonable results. A historical exception to this approach where joints designed as pinned for vertical loads were assumed to provide resistance to wind loads was developed into the wind-moment method of design[4].

If the joints between members in a frame are not fully rigid, the bending moments and deflections are influenced by the joints' rotational and shear stiffnesses. Eurocode 3 Part 8[5] includes sections on the calculation of the rotational stiffness of joints and their classification by stiffness for use in structural analysis. In the UK, the National Annex advises against the use of semi-continuous elastic design, except where it is supported by test evidence based on satisfactory performance in similar situations. This is because of a lack of confidence in the accuracy of the proposed method of determining joint stiffnesses and, consequently, in the validity of structural modelling when such joint stiffnesses are included. The traditional approach where joints are either assumed to be pinned or fully rigid is advocated.

In due course it is assumed that the models of joint stiffness will be validated by testing. At such a time, the inclusion of the rotational stiffness of joints in analysis models may well be required when considering certain kinds of structure. For example, portal frames built using cold formed sections often include joints that, by their nature, are of low stiffness. Allowance for joint flexibility in this form of structure is essential. Finite element software packages already allow the rotational stiffnesses to be included in an analysis model, but there is no hand method of calculation readily available for concept design of simple frames or for checking the impact of joint stiffness on the frame bending moments and sway deflections.

Slope-deflection equations

Readers of a certain vintage will remember that the slope-deflection (S-D) equations are developed for a beam element and relate the end rotations and relative displacement of the ends to the end moments; they can be derived from consideration of the curvature along the element see Figure 1.

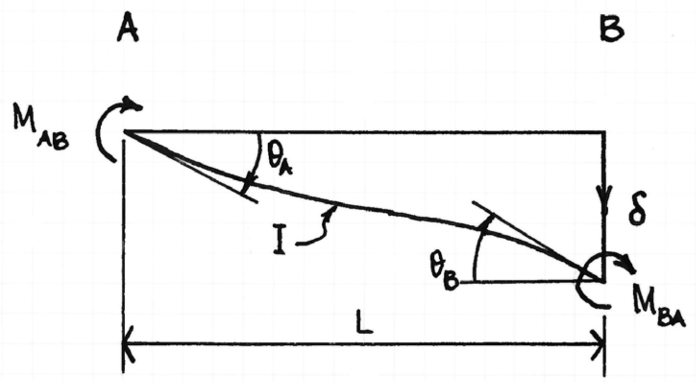


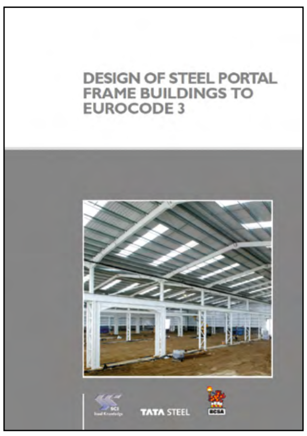
Figure 1: Deformation of beam element

For a beam element A-B, length L, second moment of area I, the equations are:

$$M_{AB} = \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\delta}{L}\right] - M_F$$

$$M_{BA} = \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\delta}{L}\right] + M_F$$

where θ_A is the rotation or slope at end A and δ is the deflection of B relative to A. M_{AB} is the bending moment at A and M_F is any fixed-end moment that may be present for a given load case[6]. Simple statically indeterminate frames can be analysed by using the slope-deflection equations to eliminate the redundancies. At a joint between two elements, the sum of the internal moments derived for each member is zero. So for elements A-B and B-C, joined at B, $M_{BA} + M_{BC} = 0$. The assumption of axially rigid members allows vertical and horizontal deflections at the ends of sloping members to be related to each other.



Modified Kleinlogel Formulae

The introduction of flexible joints means that the end rotations are increased for a given bending moment in a joint normally assumed to be rigid and reduced in a joint normally assumed to be pinned, with a corresponding redistribution of bending moments. This principle is already in use in the analysis and design of portal frames as described in SCI publication P399[7]. In the UK, portal frames are often assumed to have pinned feet in the strength analysis and this approach produces upper-bound bending moments in the columns and rafters. ▸26

▸24 However, when considering sway stability and serviceability, nominal values for the rotational stiffness of bases are usually included and this reduces eaves deflections and global second-order effects.

To avoid complicated formulae with many terms, Kleinlogel introduced simplifying notation for specific frames. For the case of a uniform load *w* applied to the rafter of a pinned-foot goal post frame of height h and span *L* (Figure 2), Kleinlogel gives the eaves (rafter to column joint) moments as:

$$M_A = M_B = -\frac{wL^2}{4N}$$

where N = 2*k* + 3 and $k = I_r/I_c.h/L$. The *I* values are those of the rafter and column, as indicated by the suffices. The same frame with fixed feet has the following eaves moments:

$$M_A = M_B = -\frac{wL^2}{6N_1}$$

The moments at the column feet are half the eaves moment:

$$M_C = M_D = \frac{wL^2}{12N_1}$$

where $N_1 = k + 2$. These formulae provide the "end cases" where the joint stiffnesses are either zero (pinned) or infinite (fixed).

The equivalent formulae to Kleinlogel's for the goal post portal frame (see Figure 2) with joint stiffness included, carrying a uniform load on the rafter can be determined as follows.

Introducing a rotational spring of stiffness k_θ at the foot of column C-A, with units kNm/radian, where the rotation is θ_C, the bending moment due to the spring is $k_{\theta C}.\theta_C$. At C, the sum of the moment in the column and the moment in the rotational spring is zero or, using the S-D equation with *h* denoting the column height,

$$k_{\theta C}\theta_C + \frac{2EI_c}{h}\left[2\theta_C + \theta_A - \frac{3\delta}{h}\right] = 0$$

For a symmetrical load case, $\delta = 0$. At the eaves joint, the slope of the rafter A-B at the joint is the sum of the rotations due to the beam stiffness and the spring stiffness. There is no deflection in the rafter because the column elements are axially rigid. Using the S-D equation:

$$\frac{M_{AB}L}{2EI_r} + \frac{M_{AB}}{k_{\theta A}} = [2\theta_A + \theta_B]$$

Solving for the bending moments, at the column feet,

$$M_{CA} = M_{DB} = \frac{wL^2}{12} \cdot \frac{(1 - 2K_C)K_A}{(2 - K_C + kK_A)}$$

At the eaves,

$$M_{AC} = M_{BD} = -\frac{wL^2}{12} \cdot \frac{(2 - K_C)K_A}{(2 - K_C + kK_A)}$$

In these formulae,

$$K_C = \frac{2EI_c}{k_{\theta C}h + 4EI_c} \; ; \; K_A = \frac{k_{\theta A}h}{k_{\theta A}h + 2kEI_c}$$

The joint stiffnesses $k_{\theta C}$ and $k_{\theta A}$ are those of the column feet and eaves

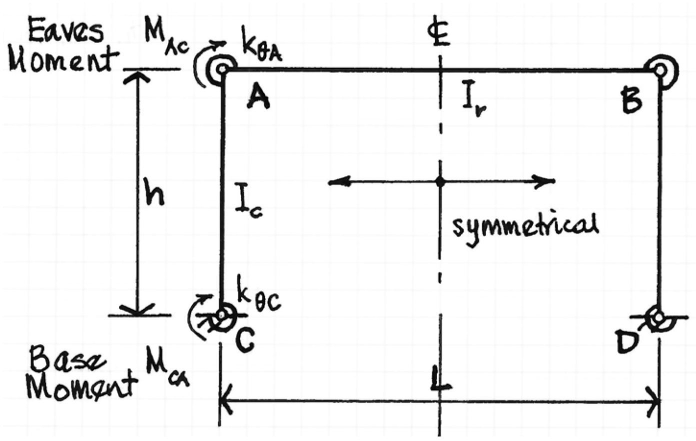


Figure 2: Goal post portal frame

respectively. For a pinned joint, the coefficients K_C and K_A equal 0.5 and 0 respectively; for a rigid joint the corresponding values are 0 and 1. Kleinlogel's formulae are reproduced when these values are substituted. A formula for eaves deflection under lateral load can also be developed.

Using this approach, simple frames with members of uniform cross-section can be analysed using the S-D equations with rotational springs at the joints. The analysis gives the same results as a stick FE analysis if the elements are chosen to exclude shear stiffness. The effect of joint stiffness can be investigated and graphs drawn using spreadsheets to show the impact on bending moment distribution around the frame, on the elastic critical load factor and on such serviceability issues as eaves spread.

Conclusions

The slope-deflection equations modified to include joint stiffness can be used effectively as a checking or investigatory tool in the design of simple frames such as goal-post or pitched portals, where joint stiffness is to be included in the design.

In their development, elements are assumed to be uniform in cross section along their length and to exhibit bending deformation only, unlike element formulations in some FE software which also include shear deformation.

In a future article, the effect of varying joint stiffness on characteristics such as frame stability and eaves spread is investigated. Spreadsheet tools to allow rapid calculations are discussed. ■

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