Verification of beams subject to a hogging bending moment

David Brown of the SCI considers the solutions to this complex situation

What's the problem?

In buildings, beams are generally designed as simply supported. Even composite beams are assumed to be simply supported - when we readily appreciate they are not. Just occasionally, designers are faced with a member where the bending moment reverses at some point within the member length. In continuous floor beams there will usually be restraints to one flange only, so there will be a length where the other flange is unrestrained. The bending moment diagram for a continuous beam is of the form shown in Figure 1.

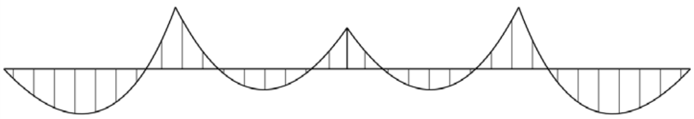


Figure 1: Continuous beam bending moment diagram

In this case, the top flange of the beam is usually restrained, possibly at intervals, but more commonly a continuous restraint to the top flange. In the hogging portions of the bending moment diagram, the bottom flange is in compression and unrestrained. The challenge is to verify this length.

"It's easy, the point of contraflexure is a restraint"

This is a common assumption, which the SCI is occasionally asked to endorse when applied to the floor beam considered in Figure 1. The idea comes from assumptions made in the design of portal frames, with the suggestion that this can be applied to floor beams. After all, the beam does not know if it is in a portal frame or in a floor.

The practice of assuming the point of contraflexure to be a "virtual lateral restraint" to the bottom flange has been enshrined in portal frame design for many years. The advice is found in clause 5.5.5 of BS 5950 and in SCI publications P252 (BS 5950) and P399 (Eurocode design). There are certain requirements to be met, which are clearly related to the idea of an "inverted U-frame", which has been covered in other *New Steel Construction* articles. The purlins must be sufficiently stiff – manifest as the rule that they must be at least 25% of the rafter depth. The connection to the flange must be sufficiently rigid – manifest as the rule that the connection from purlin to rafter must have at least two bolts. These rules had their origins in the 1970's, when purlins were hot rolled and rafters had tapered flanges. When discussing this question, Professor Horne commented "...even the small torsional restraint obtained with a continuous rail and two bolts in the cleat, but without a web stiffener, is sufficient to prevent the spread of torsional failure from a length of member with the outstand flange in compression to part of the member with the outstand flange in tension".

P399 subtly notes that the assumption of a virtual lateral restraint to the bottom flange is UK practice. Other designers may be suspicious of this bold assumption.

The suggestion is that this assumption may be applied to a floor beam which has a similar arrangement – restraint to the top flange and a bottom flange which changes from tension to compression.

Although many designers might have taken this route, verifying the member between the point of contraflexure and the support, published guidance prohibits this. In the *Designers' Guide to EN 1994-1-1*[1], section 6.4.1, we read "It should not be assumed that a point of contraflexure is equivalent to a lateral restraint".

In some situations, it is common practice to avoid the uncertainty altogether and provide a restraint at the required location. This is the typical solution for bridges, and (for example) trusses.

What are the alternatives?

In short, the answer is to "do it properly". The task is straightforward if the member is a bare steel beam. The proper approach is complicated if the member is a composite beam, so this article proposes that a conservative approach is to pretend the composite beam is in fact steel alone. There is a "simplified verification" method for composite beams in the design standard which does not involve any calculations (it does in the UK National Annex variation!) but as will be seen, the scope means it is of very limited use. The two approaches are examined in the following sections.

Bare steel beam

The solution here is to model the complete span in *LTbeam* or *LTBeamN*, ensure that the bending moment diagram is correct, model the correct restraints and use the software to determine M_{cr}. The calculation of the lateral torsional buckling resistance $M_{b,Rd}$ then follows the normal route. The resistance is checked against the largest moment in the span, which for a fixed ended beam and UDL, will be at the support.

Figure 2 shows the dimensions, loading and resulting bending moment diagram for a continuous beam with fully fixed supports. The hogging moment at the support is $wL^2/12$. The point of contraflexure is 2114 mm from each support.

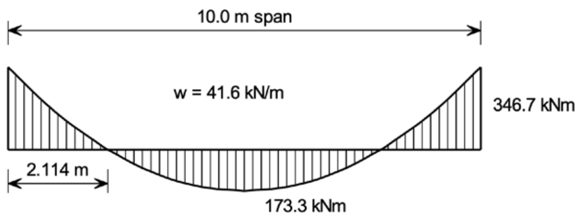


Figure 2: Beam and bending moment diagram

It is assumed that the beam is non-composite, but has a continuous lateral restraint to the top flange – presumably from whatever applies the UDL.

The beam may be modelled with fixed supports, or as simply supported but with a hogging moment applied at each end – it makes no difference to the value of M_{cr}. The selected beam is a 406 × 178 × 60, in S355. With a continuous lateral restraint to the top flange the value of M_{cr} = 1031 kNm. The buckled form is shown in Figure 3(a) (over page).

Completing the process:

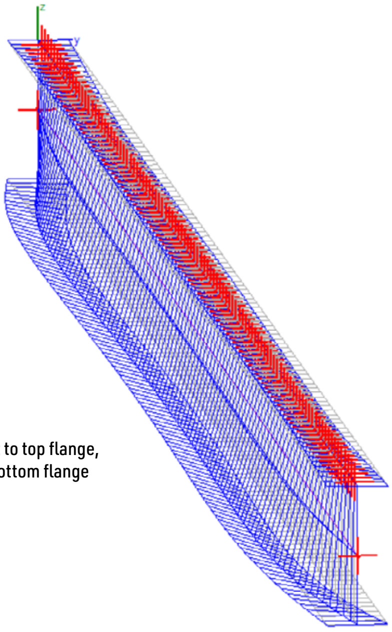
$\bar{\lambda}_{LT} = 0.643$; $\phi_{LT} = 0.714$; $\chi_{LT} = 0.861$; $f = 0.819$

$\chi_{LT,Mod} = 1.0$

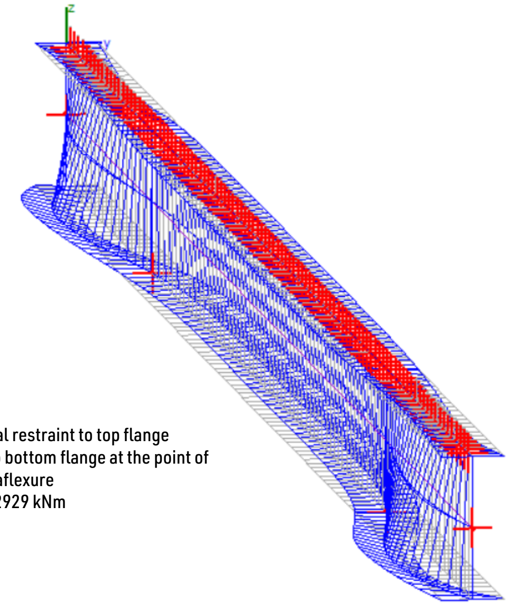
$M_{b,Rd}$ = 426 kNm, > 347 kNm, OK.

If a restraint is introduced to the bottom flange at the point of contraflexure, the buckled form is shown in Figure 3(b), and M_{cr} = 2929 kNm – quite different to the real situation.

If the prohibited approach of simply checking from the point of contraflexure to the support had been followed (for interest, not a SCI ▸26



(a) Lateral restraint to top flange, no restraint to bottom flange M_{cr} = 1031 kNm



(b) Lateral restraint to top flange and to bottom flange at the point of contraflexure M_{cr} = 2929 kNm

Figure 3: Buckled form of continuous beam

▸25 recommendation!) M_{cr} = 2426 kNm, demonstrating that the elastic buckling moment is wildly different to that of the correctly modelled beam.

Composite beams

Verification of the hogging zone of a continuous composite beam is covered in clause 6.4.2 of BS EN 1994-1-1. The principles are straightforward and familiar – a pair of beams and the slab form an inverted U-frame, as shown in Figure 4 (taken from Figure 6.11 of the standard).

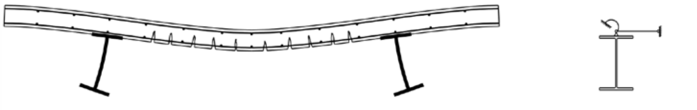


Figure 4: U-frame and model (from BS EN 1994-1-1)

The stiffness of the inverted U-frame depends on the stiffness of the slab, the stiffness of the beam web and (at least conceptually) the stiffness of the connection between beam and slab. Reference 1 notes that the flexibility of the shear connection between beam and slab can be neglected.

Once the stiffnesses have been calculated, M_{cr} for the composite member can be determined, which, as shown in Figure 4, is based on the member with a continuous lateral restraint to the top flange and a rotational spring stiffness at the same level.

Unfortunately, the process is not for the faint-hearted. Reference 1 notes that the calculation of the rotational spring stiffness is straightforward "apart from finding the cracked flexural stiffness of a composite slab". This calculation requires knowledge of the profiled slab dimensions, slab reinforcement and properties of the cracked composite section.

M_{cr} is then calculated, but this is for the composite section. The expression for a uniform steel beam cannot be used. The Eurocode does not give an expression, but this may be found in Reference 1. A value of the reduction factor χ_{LT} is calculated, but this is applied to the design resistance (in hogging) of the composite section. The resistance is compared to the hogging moment including the effects of shrinkage. In all, a complex set of calculations for designers who are not experienced in the detail of composite design – made even more complicated by the continuity which gives rise to hogging moments, the effects of cracking and shrinkage. Designers are commended to review example 6.7 in Reference 1 before undertaking their own verifications.

Simplified verification of composite beams

BS EN 1994-1-1 clause 6.4.3 offers the attractive prospect of a very much simplified approach "without direct calculation". In the core Eurocode, this is a simple test of the steel beam depth – below a tabulated maximum depth, there is no need to complete any calculation – the member is deemed to satisfy. The core Eurocode presents maximum depths for IPE sections. The UK National Annex demands the calculation of a "section parameter" in NA.2.8, which is a purely geometric parameter, but more involved than the section height limit in the core Eurocode.

This approach looks very appealing, but the associated conditions in clause 6.4.3(1) mean that in common practice, designers may be frustrated that they fall outside the scope. In addition to limitations on relative span lengths, the loading must be uniformly distributed – but critically, the design permanent load must exceed 40% of the design total load.

In the calculation which resulted in the design load of 41.6 kN/m used above, the characteristic loading was taken as g_k = 3.0 kN/m² and q_k = 5.0 kN/m², which is considered to be a reasonable pair of loads for a typical composite beam. The design loading is therefore:

Permanent: 1.35 × 3.0 = 4.05 kN/m²

Total: 1.35 × 3.0 + 1.5 × 5 = 11.55 kN/m²

The design permanent load is therefore only 35% of the design total load, so the use of the simplified approach is not permitted.

Conservative solution for composite beams

The approach proposed here may be very conservative, but it has the advantage of speed. If the bare steel beam is modelled with a lateral restraint (only) to the top flange, and found to be satisfactory, the composite member will also be satisfactory. Modelling as a bare steel beam neglects the contribution of the slab and the rotational spring stiffness.

As a comparison, consider example 6.7 in Reference 1. The verification in the hogging region concludes that the composite resistance of 767 kNm exceeds the ultimate moment with shrinkage included of 656 kNm. The steel beam is an IPE450 in S355, 12 m span and one half of a two-span continuous beam.

In *LTBeam* the loading was arranged to produce the correct hogging moment at the internal support. A continuous lateral restraint to the top flange was modelled. M_{cr} = 1098 kNm from this analysis.

Completing the process:

$\bar{\lambda}_{LT} = 0.741$; $\phi_{LT} = 0.789$; $\chi_{LT} = 0.800$; $f = 0.955$

$\chi_{LT,Mod} = 0.838$

$M_{b,Rd}$ = 505 kNm, which is unsatisfactory and shows the method to be conservative.

This result could be improved if the rotational spring stiffness was included in the model – if the rather involved calculations were undertaken to determine the stiffness. Taking a significant short cut by adopting the value of 96.4 kNm/rad calculated in example 6.7, M_{cr} increases to 2234 kNm, and $M_{b,Rd}$ = 584 kNm – still not satisfactory.

With only a lateral restraint to the top flange, an IPE500 delivers a resistance of 648 kNm, which is close enough to the 656 kNm requirement, recognising that there is benefit from the rotational spring stiffness at the top flange which has been neglected in the calculation.

Conclusions

1. The practice of assuming the point of contraflexure to be a virtual lateral restraint to the bottom flange is enshrined in the design standard for portal frames and confirmed by practice, but correctly prohibited for beams in buildings.
2. A simple buckling analysis shows that if the hogging length is assumed to be restrained at the point of contraflexure, the result is a significantly higher value of M_{cr} (i.e. an artificially high buckling resistance), and quite different to modelling the real condition.
3. If the member is bare steel, modelling the complete beam, with restraints (if any) is the straightforward and correct approach. Tools are available to calculate M_{cr}.
4. With a composite beam, the full process is complex. The codified simplified method is very limited in scope. Assuming the beam to be steel alone will be conservative.

A webinar on continuous composite beams is to be presented on 15 June: See the SCI website for details

References

[1] Johnson, R. P & Anderson, D.
Designers' Guide to EN 1994-1-1- Eurocode 4: Design of composite steel and concrete structures. Part 1.1: General rules and rules for buildings.
Thomas Telford, 2004