Design of beam-column splice connections according to Eurocode 3

Ricardo Pimentel of the SCI discusses the design of beam-column splice connections considering second-order effects due to combined flexural and lateral torsional buckling according to Eurocode 3.

Introduction

Buckling phenomena cause additional internal forces within members due to local second order effects (P-δ). Recent NSC articles [1], [2], [3] introduced these effects, giving theoretical background and practical applications. Reference [3] provides a detailed worked example of the assessment of the second order bending moment on columns due to strut action for column splices designed under pure compression. Members subjected to major axis bending that are susceptible to lateral torsional buckling are also subjected to second order effects, because the major axis bending induces a horizontal deflection (minor axis - δ_h), vertical deflection (major axis - δ_v) and a cross-sectional rotation (θ) as illustrated in Figure 1. Such deformations will increase as the applied bending moment increases. When the bending moment is close to the so-called elastic critical moment, the deformation increases rapidly and failure occurs.

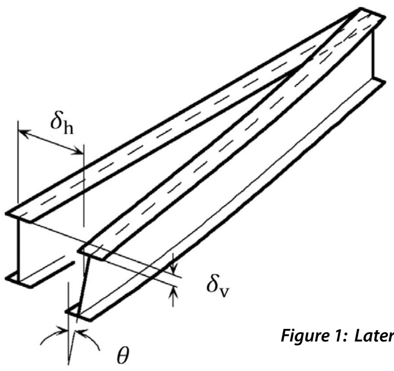

Figure 1: Lateral torsional buckling mode shape

Addressing second order effects

Whilst for a strut an equivalent initial bow imperfection can be back-calculated relatively easily and amplified to account for the second order effect, the problem for lateral torsional buckling phenomena offers a much more complex challenge. Although the effects of the vertical displacement and rotation have an impact on the lateral torsional buckling resistance of the member, the consideration of an equivalent horizontal out of plane bow imperfection offers a good approximation to establish the initial member imperfection. EN 1993-1-1 clause 5.3.4 (3) supports this approach. A precise analysis including the amplification of the initial member imperfection is complex and usually undertaken by numerical analysis with advanced finite element model tools. A reasonable approximation can be achieved by manual methods, as demonstrated in this article. The process described is useful when designing splice connections in unrestrained beams.

Lateral torsional buckling failure criteria

The design buckling resistances for buckling phenomena according to Eurocode philosophy are calibrated based on an elastic cross section failure, where all imperfections (such as residual stresses, lack of straightness, etc.) are accounted for by an equivalent imperfection factor α. Second order local effects are implicitly considered by the Eurocode design method (section 6.3). Reference [1] explains this concept for a strut. Using the same principles for an element subjected to lateral torsional buckling, the buckling failure can be understood as a critical stress, for which two components can be identified: (i) component due to major axis bending (σ_{M_y}); (ii) component due to the second order bending moment under minor axis bending ($\sigma_{M_{z,P\delta,LTB}}$).

Out of plane bending moment due to lateral torsional buckling

If the buckling failure is considered as an elastic cross section failure (with a material yield strength of f_y), the following condition can be established:

$$f_y = \sigma_{M_y} + \sigma_{M_{z,P\delta,LTB}}$$

According to Eurocode nomenclature, the buckling resistance can be established as the product of the reduction factor for buckling phenomenon χ multiplied by the design characteristic resistance. As the characteristic resistance is directly proportional to the material resistance, the stress at lateral torsional buckling failure can be established as $\chi_{LT} \cdot f_y$ (described as the critical buckling stress). The stress $\sigma_{M_{z,P\delta,LTB}}$ can be defined based on cross section properties and the second order bending moment $M_{z,P\delta,LTB}$, which leads to:

$$f_y = \chi_{LT} \cdot f_y + \frac{M_{z,P\delta,LTB}}{W_{el,z}}$$

Dividing the previous equation by the critical buckling stress, it can be demonstrated that:

$$\frac{f_y}{\chi_{LT} \cdot f_y} = \frac{\chi_{LT}}{\chi_{LT} \cdot f_y} \cdot f_y + \frac{M_{z,P\delta,LTB}}{\chi_{LT} \cdot f_y \cdot W_{el,z}} \leftrightarrow M_{z,P\delta,LTB} = \left(\frac{1}{\chi_{LT}} - 1\right) \cdot \chi_{LT} \cdot M_{z,el,Rk}$$

Where $M_{z,el,Rk}$ is the out of place elastic bending resistance of the cross section.

According to the Eurocode definition, χ_{LT} is the ratio between the buckling bending resistance and the characteristic bending resistance of the cross section. As the buckling bending resistance ($M_{b,Rd}$) should be always less than the applied bending moment ($M_{y,Ed}$), it can be approximately (and conservatively) assumed that:

$$\chi_{LT} = \frac{M_{b,Rd} \cdot \gamma_{M1}}{M_{y,el,Rk}} \approx \chi_{LT} = \frac{M_{y,Ed} \cdot \gamma_{M1}}{M_{y,el,Rk}}$$

Where γ_{M1} is that partial factor for buckling phenomenon according to the UK NA to BS EN 1993-1-1 [4].

This leads to:

$$M_{z,P\delta,LTB} = \left(\frac{1}{\chi_{LT}} - 1\right) \cdot \frac{M_{z,el,Rk}}{M_{z,el,Rk}} \cdot M_{y,Ed} \cdot \gamma_{M1} \qquad \text{Eq (1)}$$

The complexity of the procedure is related to the calculation of χ_{LT}. For cases where section 6.3.2.3 (2) of EN 1993-1-1 is applied, $M_{z,P\delta,LTB}$ should be multiplied by "f".

Splices of elements under compression

Splices subjected to axial compression should be designed for the following forces:

1. N_{Ed} – Applied axial force;
2. $M_{i,P\delta,FB}$ – Second order bending moment due to strut action (flexural buckling) about the axis "i".

It should be clear that a member only experiences flexural buckling under one of its axes. The design bending moments $M_{i,P\delta,FB}$ should be only considered about the weak axis for flexural buckling (i.e. the axis which shows the higher slenderness – reflected in a higher value of $\bar{\lambda}$ - according to EN 1993-1-1 section 6.3.1.2).

The second order bending moment due to strut action can be calculated as follows:

$$M_{i,P\delta,FB} = N_{Ed} \cdot e_{P\delta,i} = N_{Ed} \cdot e_{0,i} \cdot k_{amp,i} \cdot \gamma_{M1} \qquad \text{Eq. (2)}$$

Where:

N_{Ed} is the applied axial load;

$e_{0,i}$ is the initial bow imperfection about axis "i" equal to $\frac{W_{el,i}}{A}\,\alpha\,(\bar{\lambda}_i - 0.20)$;

▶38

▶37

$e_{P\delta,i}$ is the bow imperfection accounting for the second order effects;

$k_{amp,i}$ is the amplification factor equal to $\frac{N_{cr,i}}{N_{cr,i} - N_{Ed}}$;

$W_{el,i}$ is the elastic modulus of the cross section about axis "i";

A is the cross-section area;

α is the equivalent imperfection factor according to EN 1993-1-1 section 6.3.1.2;

$\bar{\lambda}_i$ is the non-dimensional slenderness according to EN 1993-1-1 section 6.3.1.2 about axis "i";

$N_{cr,i}$ is the elastic critical buckling load for flexural buckling under the axis "i": $N_{cr,1} = \frac{\pi^2 EI_i}{L_{crit,i}}$, where I_i is the second moment of area about axis "i" and $L_{crit,i}$ is the buckling length about axis "i";

N_{Ed} is the applied axial load on the column.

Splices of elements under bending

Splices within unrestrained segments subjected to major axis bending should be designed for the following forces:

1. $M_{Ed,y}$ – Applied bending moment under the major axis;
2. $M_{Ed,z}$ – Applied bending moment under the minor axis;
3. $M_{z,P\delta,LTB}$ – Second order bending moment due to lateral torsional buckling.

Beam-column splices

Beam-column splices can be exposed to the following design forces:

1. N_{Ed} – Applied axial force;
2. $M_{Ed,y}$ – Applied bending moment about the major axis;
3. $M_{Ed,z}$ – Applied bending moment about the minor axis;
4. $M_{i,P\delta,FB}$ – Second order bending moment due to strut action (flexural buckling) about the axis "i";
5. $M_{z,P\delta,LTB}$ – Second order bending moment due to lateral torsional buckling;
6. $M_{i,P\delta,Amp}$ – Moments due to the amplification of the applied bending moments due to the strut action about the axis "i".

As for elements under compression, the design bending moments $M_{i,P\delta,FB}$ should be only considered about one of the cross-sectional axes for flexural buckling. Beam-columns experience an additional bending moment $M_{i,P\delta,Amp}$ which is related to the amplification of the applied bending moments due to the presence of axial load. The second order bending moments due to the presence of axial force can be calculated considering the amplification factor about the axis "i" as follows:

$$M_{i,P\delta,Amp} = M_{Ed,i} \cdot \left[\frac{N_{cr,i}}{N_{cr,i} - N_{Ed}} - 1 \right] \quad \text{Eq. (3)}$$

The minor axis bending moment $M_{z,P\delta,Amp}$ should be always considered. The effects of $M_{y,P\delta,Amp}$ and $M_{z,P\delta,Amp}$ should not be considered together: designers should consider two independent combination of action where $M_{y,P\delta,Amp}$ or $M_{z,P\delta,Amp}$ are considered. This is because the second order effects will only develop about one of the member axes, i.e. either LTB will govern and the beam will deform sideways, or a major axis second order bending moment will be generated.

The procedure described above comprises segments under a uniform bending moment profile along the segment. To assess other bending moment profiles, designers may consider the value of $C_{m,i}$ from EN 1993-1-1 Table B.3. For such cases, the values of $M_{i,P\delta,Amp}$ obtained from equation 3 may be multiplied by the values of $C_{m,i}$.

As a summary, the design forces for a beam-column splice can be established by the following equations:

$$N_{Ed,splice} = N_{Ed} \quad \text{Eq. (4)}$$

$$M_{Ed,y,splice} = M_{Ed,y} + [M_{y,P\delta,FB}] + \{M_{y,P\delta,Amp}\} \quad \text{Eq. (5)}$$

$$M_{Ed,z,splice} = M_{Ed,z} + [M_{z,P\delta,FB}] + \{M_{z,P\delta,LTB}\} + M_{z,P\delta,Amp} \quad \text{Eq. (6)}$$

Pairs of effects within the square and round brackets should not be considered simultaneously. Designers should consider them individually and assess which combination of forces gives the most onerous design condition.

Second order bending moment distribution along an unrestrained segment

The bending moment diagrams calculated according to equations 1, 2 and 3 represent a maximum value at mid span of an unrestrained segment. The second order bending moments follow a sinusoidal shape between points of inflexion (points between which the effective length is measured) of: $M_{i,P\delta}(x) = M_{i,P\delta,max} \cdot \sin(\pi{\cdot}x / l)$, where "$x$" is the position from a point of inflexion and "l" is the length between points of inflexion (for a pinned column, this is the column length).

Comparison with BS 5950 approach

Previous UK practice design addressed second order effects for columns, beams and beam-column splices according to BS 5950 [6]. Further guidance was given by SCI AD notes 243 [7] and AD 244 [8].

The second order out of plane bending moment is addressed by BS 5950 Annex B.3. While BS 5950 established the second order bending moment based on a relationship between yield strength and bending strength for lateral torsional buckling, the Eurocode nomenclature establishes it based on the parameter χ_{LT}. The parameter χ_{LT} can also be understood as a relationship between the allowable buckling stress and the yield strength. Therefore, $1 / \chi_{LT}$ represents the same relationship as proposed by BS 5950. The factor m_{LT}, which considers the bending moment diagram shape along the segment, is accounted for while calculating χ_{LT} according to EN 1993-1-1 6.3.2 (within the elastic critical bending moment - M_{cr}). Both BS 5950 and Eurocode 3 approach have the same background.

Strut action is defined by Annex C.3 of BS 5950. Both BS 5950 and EN 1993-1-1 approaches to address flexural buckling are based on an elastic cross section failure due to the combined stresses of axial load and second order bending moments due to the strut action. If the same buckling resistances are assumed, and considering the elastic section modulus, the simplified method from Annex C.3 of BS 5950 tends to give conservative values in comparison with equation 2. A similar answer for the strut moment is obtained if the applied load is close to the buckling resistance.

Second order effects for members subjected to combined axial load and bending are defined by Annex I.5 of BS 5950. The expression $1/(p_{Ei}/f_c - 1)$ gives the same answer as $[(N_{cr,i}/(N_{cr,i} - N_{Ed}) - 1]$ if the same buckling resistances are assumed. The values of m_y and m_x according to BS 5950-1 Annex I.5 (which should be defined according to BS 5950-1 4.8.3.3.4) are similar to the values defined by EN 1993-1-1 Table B.3.

Calculation example

Consider a UB 533 × 165 × 66 beam-column element with an unrestrained segment of 5 m length subjected to an axial load of 150 kN and a linear bending moment diagram between 165 kNm and 82.5 kNm. A splice connection is located at 1/3 (1.67m) of the unrestrained segment length, closer to the point of maximum bending moment. The bending moment at the splice location is therefore 137.5 kNm. The calculation of the second order design forces to design the splice connection is summarized in the table below. Member resistances are taken from the Blue Book.

Conclusions

1. Lateral torsional buckling failure can be considered by means of an equivalent initial horizontal bow imperfection under the minor axis of the profile, which must then be amplified;
2. Considering the member lateral torsional buckling capacity, it is possible to estimate the cross-section forces at failure;
3. The failure criteria for lateral torsional buckling is assumed to be elastic failure of the cross section considering major axis bending and the second order bending moment due to lateral torsional buckling; strut action effects also need to be accounted for in beam-columns;
4. EN 1993-1-1 approaches for beam and beam-column splices follow the same principles as BS 5950.

References

1 Pimentel, R., Stability and second order of steel structures: Part 1: fundamental behaviour; New Steel Construction; vol 27 No 3 March 2019;
2 Pimentel, R., Stability and second order of steel structures: Part 2: design according to Eurocode 3; New Steel Construction; vol 27 No 4 April 2019;
3 Eurocode 3 - Design of steel structures - Part 1-1: General rules and rules for buildings; BSI, 2014;
4 NA BS EN 1993-1-1+A1 UK National Annex to Eurocode 3 - Eurocode 3 - Design of steel structures - Part 1-1: General rules and rules for buildings; BSI, 2014;
5 Henderson, R., Bearing splice in a column; New Steel Construction; vol 28 No 3 March 2020;
6 BS 5950, Structural use of steelwork in building: Part 1: Code of practice for design - Rolled and welded sections, BSI, 2000;
7 SCI Advisory Desk Notes: AD 243: Splices within unrestrained lengths;
8 SCI Advisory Desk Notes: AD 244: Second order moments

Section properties and resistances, critical loads (S355); UB 533 × 165 × 66	Eurocode buckling Resistances	EN 1993-1-1 P-δ effects	Critical design effects for splice design
$A = 83.7\ cm^2$	$N_{b,rd,y} = 2890$ kN	$k_{amp,y} = 1.005$	$N_{Ed,splice} = N_{Ed} = 150$ kN
$W_{el,y} = 1340\ cm^3$	$N_{b,rd,z} = 598$ kN	$k_{amp,z} = 1.267$	$M_{Ed,y,splice} = M_{Ed,y} = 137.5$ kNm
$W_{el,z} = 104\ cm^3$	$M_{b,rd} = 225$ kNm	$\bar{\lambda}_z = 2.04$	$M_{Ed,z,splice} = M_{z,P\delta,FB} + M_{z,P\delta,LTB}$ $M_{Ed,z,splice} = 1.3 + 16.2 = \pm 17.5$ kNm
$W_{pl,y} = 1560\ cm^3$	Note: $C_1 \approx 1.35$	$e_{0,z} = 7.8$ mm ($\alpha = 0.34$)	The set of design actions presented above give the most onerous design scenario according to equations 5 and 6.
$W_{pl,z} = 166\ cm^3$		$e_{P\delta,z} = 9.9$ mm	
$I_y = 35000\ cm^4$		$M_{z,P\delta,FB,max} = 1.5$ kNm	
$I_y = 859\ cm^4$		$M_{z,P\delta,FB}$ (@ 1.67 m) = 1.3 kNm	
$M_{y,pl,Rd} = 554$ kNm		$M_{z,P\delta,LTB,max} = 18.7$ kNm	Note: the value of χ_{LT} was calculated as $M_{b,rd} / M_{y,rd} = 225/554 = 0.41$
$M_{z,pl,Rd} = 59$ kNm		$M_{z,P\delta,LTB}$ (@ 1.67 m) = 16.2 kNm	
$M_{y,el,Rd} = 474$ kNm		$M_{y,P\delta,Amp,max} = 0.86$ kNm	
$M_{z,el,Rd} = 36.9$ kNm		$M_{y,P\delta,Amp}$ (@ 1.67 m) = 0.74 kNm	
$N_{cr,y} = 29017$ kN		($C_{m,y}$ is assumed as 1 considering the low value of $M_{y,P\delta,Amp,max}$)	
$N_{cr,z} = 712$ kN			

Design forces and bending moments for splice design