Joint stiffness and the elastic critical load factor

The susceptibility of moment-resisting frames to global buckling is profoundly influenced by the stiffness of joints as calculated by the proposed method in BS EN 1993-1-8. Richard Henderson of the SCI illustrates the potential effects.

1 Introduction

It is unfortunate that BS EN 1993-1-1 does not adopt a succinct label for such an important parameter as α_{cr}, the authors instead choosing the descriptor "factor by which the design loads would have to be increased to cause elastic instability in a global mode". The BS 5950 label in the title above has much greater utility. The article on the calculation of joint stiffness in the February edition of *New Steel Construction* hinted at the effect on stability of the stiffness of bolted joints and the present article provides an illustrative example.

2 Example portal frame

2.1 Rigid joints

The structure used in this simple example is a pinned-foot portal frame with a horizontal rafter. Sufficient restraints are assumed to be provided to prevent out of plane and lateral torsional buckling. The frame has a span L of 30 m and a height h to the centre-line of the rafter of 15 m. The rafter is subject to a uniform load of 10 kN/m. In order to achieve a high elastic critical load factor, stiff UB columns have been adopted, consisting of 914 × 305 UB 224 rolled sections. The rafter is a 533 × 210 UB 101. Hand analysis has been carried out for amusement and checked by stick FE analysis, first assuming the joints are infinitely stiff.

2.2 Frame deflections

For the vertical load case, determining the bending moments by moment distribution requires the stiffness coefficients for the members at the joint. Assuming symmetry, these are $k_c = 3EI_c/h$ for the column and $k_b = 2EI_b/L$ for the rafter. The distribution coefficient for the column is given by $k_c/(k_b + k_c)$. No redistribution is required and the results are obtained directly as shown in Table 2.1.

Element	I value (m⁴)	Stiffness (kNm/rad)	Distribution coefficient	FEM (kNm)	Bending moment (kNm)
Column	3.76e-3	157920	0.9483	-	+711.2
Beam	6.15e-4	8610	0.0517	-750	-711.2

Table 2.1 Vertical load case: bending moments

The free bending moment in the rafter is 1125 kNm giving a mid-span moment of 413.8 kNm. The bending resistance of the rafter cross section given in the Blue Book is 901 kNm. The mid-span deflection of the rafter is given by the difference between the simply supported deflection and the upward deflection due to the end moments:

$$\delta = \left(\frac{5wL^4}{384EI_b} - \frac{M_0L^2}{8EI_b}\right) = 0.197\text{m}$$

The lateral deflection of the frame from a horizontal load at rafter level can be found using the slope-deflection equations and is given by:

$$\delta = \frac{Hh^2}{EI_bI_c}\left(LI_c + 2hI_b\right)$$

Assuming a unit load of H = 100 kN, substituting values gives a horizontal deflection at rafter level of 0.507 m.

2.3 Elastic critical load factor α_{cr}

Using the formula in para. 5.2.1(4) of BS EN 1993-1-1,

$$\alpha_{cr} = \left(\frac{H_{Ed}}{V_{Ed}}\right)\left(\frac{h}{\delta_{H,Ed}}\right)$$

the α_{cr} value for the frame can be calculated. The global stiffness of the frame (H/δ) is 100/0.507 = 197 kN/m. Substituting the remaining values gives $\alpha_{cr} = 9.9$. According to para. 5.2.1(3) of EC3, the frame is therefore almost stiff enough for second order effects to be ignored. Increasing the rafter by one serial size would achieve this, with $\alpha_{cr} = 10.6$

2.4 Introducing joint flexibility

According to Para 5.1.2 of BS EN 1993-1-8, for elastic global analysis, joints should be classified according to their rotational stiffness. If the joint is semi-rigid, the rotational stiffness S_j corresponding to the design bending moment should be used in the analysis. A reasonable idea of the joint stiffness is therefore required to model the structure. The joint must be classified according to BS EN 1993-1-8 para. 5.2.2 and the initial rotational stiffness is denoted $S_{j,ini}$. The joint is deemed semi-rigid if:

$0.5EI_b / L \le S_{j,ini} \le 25EI_b / L$ or if $K_b / K_c < 0.1$

For the purpose of this example, the rotational stiffness of the beam to column joint has been assumed to have the same value as that calculated in February's technical article on the calculation of joint stiffness. The beam in that case was also a 533 deep UB and the joint stiffness calculated: $S_{j,ini} = k_{\theta B} = 100$ MNm/radian. Using Table 2.1, $K_b / K_c = 0.082$ but $25EI_b / L = 107$ MNm/radian so the joint is semi-rigid.

Joint flexibility increases the lateral deflection of the frame because, in addition to the rotation of the intersection of the members due to their curvature, the joints themselves rotate. The effect of this joint flexibility on the lateral deflection can be determined by assuming the joints behave as rotational springs and the members are rigid.

A similar approach to the slope-deflection equations results in the following formula for lateral deflection due to flexible joints:

$$\delta = \frac{-Hh^2}{(k_{\theta A} + k_{\theta B})}$$

Here H is the shear force in the element in kN and the k_θ

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parameters are the rotational stiffnesses in kNm/radian at ends A and B of the column of length *h* m and the deflection is in metres. This deflection is added to the deflection due to element flexure already calculated. The foot of the column is pinned so the rotational stiffness at this end is zero. Substituting the values $H = 100$ kN and $h = 15$ m gives $\delta = 0.113$ m and a total lateral deflection of 0.620 m. The revised global stiffness of the frame is 161 kN/m and the elastic critical load factor reduces to 8.07 - second-order effects must therefore be considered.

The effect of the joint stiffness on the moments and deflections due to vertical loads can be calculated by considering rotations at the joint. The slope in the rafter is equal to the simply-supported value reduced by the slope due to the end moment. This rotation is equal to the slope in the column plus the rotation due to the flexibility of the joint.

$$\frac{wL^2}{24EI_b} - \frac{ML}{2EI_b} = \frac{Mh}{3EI_c} - \frac{M}{k_\theta}$$

The value of *M* is 657.4 kNm, a reduction of 53.7 kNm. The mid-span moment increases to 467.5 kNm and the mid-span deflection to 0.244 m.

The results can be confirmed by FE analysis, assuming elements have infinite shear stiffness.

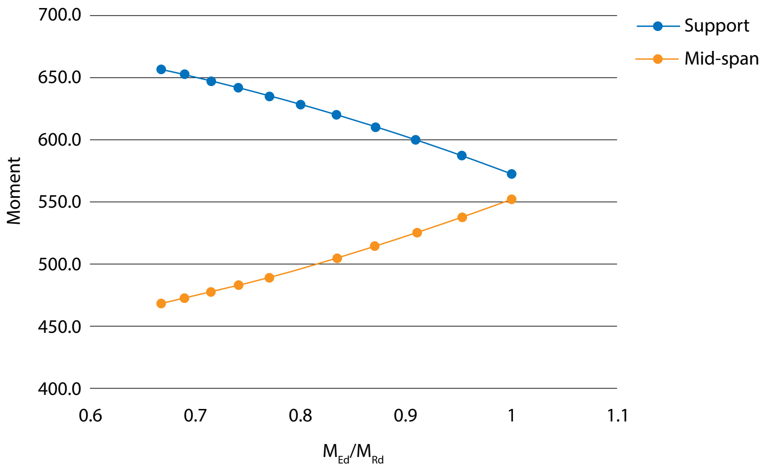

Figure 2.1 Beam Moments

2.5 Effect of connection design resistance

According to BS EN 1993-1-8 para. 6.3.1, the rotational stiffness of a beam-to-column joint $S_{j,ini}$ is reduced by a factor μ that depends on the joint utilization. If the design resistance is at least 1.5 times the design bending moment, the initial stiffness of the joint can be used in the analysis and $\mu = 1.0$. If the resistance ratio is less than 1.5 times, plastic deformation is assumed and a reduced stiffness must be used.

$\mu = (1.5M_{j,Ed} / M_{j,Rd})^\psi$ where $\psi = 2.7$ for a bolted end plate. The effect of the resistance ratio on the bending moments is shown in Figure 2.1

The support (hogging) moment reduces as the margin of resistance of the joint reduces. When the joint resistance equals the design moment, the support moment has reduced from 711 kNm, the value found from classical analysis, to 572 kNm. The mid-span (sagging) moment increases correspondingly.

The effect on the elastic critical load factor is shown in Figure 2.2. The reduction in α_{cr} with reducing joint over-design is almost linear, from 8.07 to 5.93. This reduced value would require the design lateral loads to be increased by 20% to allow for second-order effects. Increasing the rafter by one serial size gives $\alpha_{cr} = 6.2$ and a lateral load increase of 19%.

An unverified estimate of initial stiffness for the specific elements in the example found a value of about 60 MNm/radian for a joint with a moment resistance of 622 kNm. This gives $\alpha_{cr} = 6.6$ and a support moment of 603 kNm. The value of M_{Ed}/M_{Rd} is therefore 0.97 and $\mu = 2.6$ approximately. This value of μ corresponds to a lower joint stiffness which reduces the support moment to about 508 kNm. Iteration indicates a joint stiffness of about 29 MNm/radian giving a support moment of 538 kNm and $\mu \approx 2.05$. The corresponding value of α_{cr} is about 5.3.

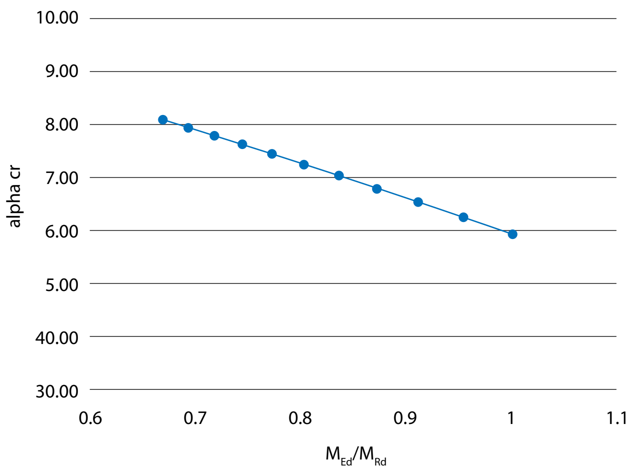

Figure 2.2 Elastic critical load factor

3 Conclusions

The above example illustrates the effect of joint stiffness on frame behaviour, in terms of the design bending moments, the deflections and the global stability and second-order effects. The sequencing of analysis and design steps is also affected as the designer must either have a preliminary idea of joint details when setting up the analysis model or iteration will be necessary.

The presence of the resistance ratio μ in the stiffness calculation potentially introduces difficulties where the frame and joints are designed by different parties. The designer could specify design moments 50% larger than those determined in the analysis, in the hope of the joint remaining elastic. The steelwork contractor could well find it challenging and expensive to satisfy such a requirement.

The UK National Annex to BS EN 1993-1-8:2005 states in clause NA.2.6 that connections designed in accordance with the principles given in the SCI publication P207[1] may be classified on the basis of the guidance given in section 2.5 of the same publication. SCI publication P398[2], the successor to P207 contains the advice that well-proportioned connections that follow the recommendations for standardisation given in P398 and designed for strength alone can generally be assumed to be rigid for joints in braced frames and single-storey portal frames.

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1 SCI P207, Joints in Steel Construction – Moment Connections

2 SCI P398 Joints in Steel Construction – Moment Resisting Joints to Eurocode 3