Stability and second order effects on steel structures: Part 2: design according to Eurocode 3

Ricardo Pimentel of the SCI illustrates the different methods provided by EN 1993-1-1 to address the topics of member stability, global frame stability and second order effects. Fundamental structural mechanics relating to stability was covered in Part 1.

Section 5.2 of EN 1993-1-1[1] introduces an approximate method to calculate the critical factor of frames (α_{cr}), based on the well-known Horne method[2] (Figure 1). The method is limited to frames with low axial force in the beams/rafters ($N_{Ed} \leq 0.10\ N_{cr,R}$; N_{Ed} is the design axial load; $N_{cr,R}$ is the elastic critical load for buckling about the major axis of the beam/rafter) and for frames not steeper than 26°. For other cases, further guidance can be found in reference 3.

In section 5.2.2 of EN 1993-1-1, different methods are proposed to consider local (*P-δ*) and global (*P-Δ*) second order effects for structural analysis and member verifications. The following three main methods can be identified:

Method 1:
Both *P-δ* and *P-Δ* effects in addition to local and global imperfections are directly considered in the global analysis; the deformed structural shape is considered in the analysis, due to local and global imperfections and local and global second order effects; second order design internal forces are calculated. This design method may need to include in-plane and out of plane flexural buckling in addition to lateral torsional buckling.

Method 2:
P-Δ second order effects and global imperfections are considered in the structural analysis; *P-δ* effects are allowed for while performing stability checks according to EN 1993-1-1 section 6.3; the deformed structural shape is considered; second order design internal forces are calculated.

Method 3:
Both *P-δ* and *P-Δ* effects are accounted for when performing stability checks according to section 6.3 of EN 1993-1-1. In this method, an equivalent member length (effective length) needs to be defined. The allowance for *P-Δ* effects is made by increasing the *P-δ* effects by means of a longer member length. First order internal forces are considered for the member verification, which may include global imperfections – see EN1993-1-1 5.3.2 (4). Global imperfections need to be included in the analysis, generally by applying the Equivalent Horizontal Forces (*EHF*). Buckling lengths greater than $2l$ may be required to allow for *P-Δ* effects in structures sensitive to those effects.

For **Method 1**, different approaches may be taken, as out-of-plane flexural buckling (FB) and lateral torsional buckling (LTB) may or may not be relevant. To allow for LTB, according to EN 1993-1-1 section 5.3.4, an equivalent bow imperfection equal to $k{\cdot}e_{0,d}$ may be used, where $e_{0,d}$ is the equivalent bow imperfection of the weak axis of the profile and k is a correction factor; it is also stated that in general, torsion imperfections need not to be considered. According to the UK National Annex[4], the value of k is to be taken as 1. The application of Method 1 is more often used in research, but several commercial software packages already allow users to directly consider the *P-δ* and *P-Δ* effects within the structural analysis. Method 1, where local and global imperfection are directly considered in the analysis, is necessary for the cases where the following condition are met (clause 5.3.2 (6) of EN 1993-1-1):

- $\alpha_{cr} < 10$, for elastic global analysis;
- At least one moment resisting joint at one member end;
- $N_{Ed} > 0.25\ N_{cr,0}$, where N_{Ed} is the design axial load and $N_{cr,0}$ the critical load assuming a pin-ended strut. This means that for a simple column system, $\alpha_{cr} = \dfrac{N_{cr,o}}{N_{Ed}} < 4$.

Method 2 can be implemented by two possible approaches:

- Method 2.1 - Considering the *P-Δ* effects directly through a numerical geometric non-linear global analysis considering global imperfections; usually computed by commercial software packages; this may increase the required analysis time for large frames and multiple load combinations;
- Method 2.2 – Considering the *P-Δ* effects indirectly by amplifying the first order sway effects (including global imperfections) by the so-called amplification factor $k_{sw} = \dfrac{1}{1\text{-}1/\alpha_{cr}}$. As introduced in Part 1[5], this method is limited to the cases where $\alpha_{cr} \geq 3$. For multi-storey buildings, the rule may be used when vertical and horizontal loads and frame stiffness are similar between storeys – see EN 1993-1-1 5.2.2 (6) B.

Both methods 2.1 and 2.2 are extensively used in practice. When verifying members according to EN 1993-1-1 section 6.3, system length should be used as the buckling length.

$$\alpha_{cr} = \left(\frac{H_{Ed}}{V_{Ed}}\right)\left(\frac{h}{\delta_{H,Ed}}\right)$$

H_{Ed} – Total storey shear;

V_{Ed} – Total vertical load at that storey;

h – Storey height;

$\delta_{H,Ed}$ – Horizontal displacement of the top storey relative to the bottom storey due to horizontal loads;

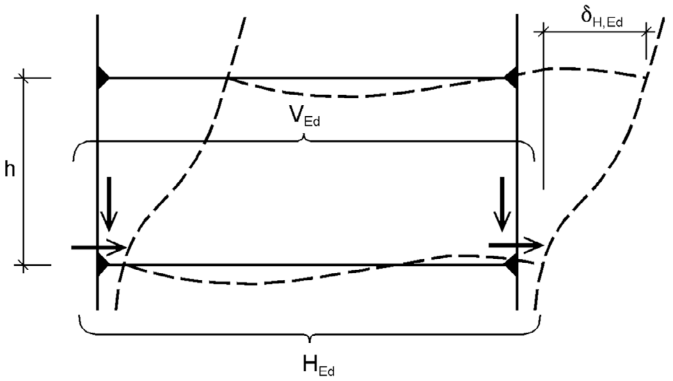
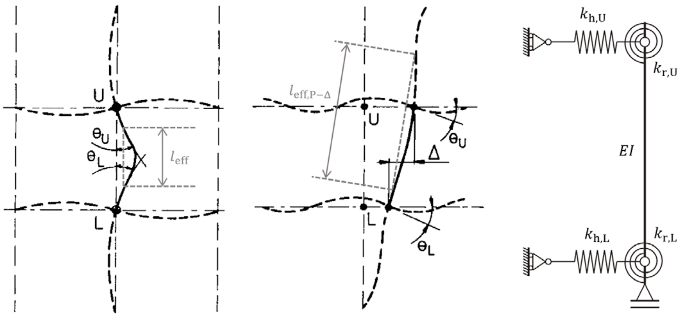


Figure 1 – Horne method to calculated α_{cr} of frames.

▶24 In **Method 3**, the designer must determine an appropriate effective length that allows for the consideration of *P-Δ* effects while performing member checks according to section 6.3 of EN 1993-1-1. As the design is based on first order internal forces, the complexity of the analysis is removed, but the effective length needs to be specified for each column. The concept of effective length was introduced in Part 1 of the current article for isolated struts, where the horizontal or rotational restraints of the strut ends were assumed as infinitely rigid. This does not represent reality: (i) rotational stiffness of the nodes is related to the flexural stiffness of the elements that are connected to the nodes, resulting in a rotational spring on each node – $k_{r,i}$ (Figure 2); (ii) if a structure is susceptible to second order global effects, the complexity is increased, as the structure is horizontally flexible (assessed by the value of α_{cr}), resulting in horizontal springs on each node – $k_{h,i}$ (Figure 2).

When a column is integrated in a frame, the concept of effective length may be described as the fictional pin-ended strut length that buckles at the same time as the frame for a specified load case[6]. Based on the value of α_{cr} for the entire frame, the critical load N_{cr} for each column can be calculated by multiplying the design axial load on each column by the value of α_{cr}. The effective lengths can then be obtained by a back calculation, knowing that $N_{cr}=(\pi^2 EI)/(l_{eff})^2$. Thus, the effective length of a column is dependent on the applied load and spring stiffness at the nodes. The values of l_{eff} obtained are only appropriate within the load arrangement assumed to calculate α_{cr}. This method is described in Annex E.6 of BS 5950-1[7].



a) Non-sway frame[8] $\alpha_{cr} \geq 10$ for elastic global analysis | b) Sway frame[8] $\alpha_{cr} < 10$ for elastic global analysis | c) Equivalent isolated column model

Figure 2: Effective length concept in sway and non-sway frames

In practice, while using **Method 3**, the definition of the effective buckling length is often obtained indirectly by a simplified analysis where each column is considered individually, with no dependency on the applied load. There are several resources to assess the problem, such as the well-known Wood method[9], which provides effective buckling lengths for sway or non-sway frames. These approximate methods are intended to provide an answer for the problem shown Figure 2c. The Wood method can be found in Annex E of BS 5950-1 as well as in NCCI SN008a[10]. Based on the model in Figure 2c, simplified methods usually assume that $k_{h,L} = \infty$ and $k_{h,U} = 0$[9].

The approximate methods provide exact results if every member has the same rigidity parameter $\varnothing_r=\sqrt{EI/N_{Ed}l^2}$ where EI is the flexural stiffness of the column, N_{Ed} is the design axial load on the column, and l is the system length of the column[6]. This means that all columns would buckle at the same time. The columns with low values of $\varnothing_r$ are the critical members (members which induce frame instability), for which the method gives conservative values of the buckling length. For members with high values of $\varnothing_r$, buckling lengths are unconservative. For the critical members, the method can be seen as a conservative approximation for the critical load of the frame[6].

The approximate methods provide an efficient and systematic procedure to assess the problem. However, the following effects/simplifications are usually disregarded/considered in the process[6,8,9,11,12]: (i) only columns are affected by *P-Δ* effects, while internal forces to design other elements (beams, connections) will be always based on first order theory; (ii) for frames sensitive to second order effects, the effective lengths calculated are the same for any value of α_{cr}; (iii) there is no influence of the applied load; (iv) for columns in non-sway frames, the rotation at opposite ends of the restraining elements are equal in magnitude and opposite in direction, producing single curvature bending; (v) for columns in sway frames, the rotation at opposite ends of the restraining elements are equal in magnitude and opposite in direction, producing double curvature bending; (vi) all columns buckle simultaneously; (vii) stiffness parameter $\varnothing_r$ is the same for all columns; (viii) no significant axial force exists in the beams; (ix) all joints are rigid; (x) joint restraint is distributed to the column above and below the joint in proportion to EI/l for the two columns. Further information about approximate methods can be found in reference 11.

Two worked examples follow, where the results obtained from the application of methods 2.1, 2.2 and 3 are compared.

Worked example 1: simple column

Influence of the number of finite elements on simple struts (Table 1):

The results support the conclusions from Part 1: for low values of N_{Ed}/N_{cr} the errors in using an approximate stiffness matrix are less significant than for cases where N_{Ed}/N_{cr} is close to 4. The consideration of 3 finite elements for the strut gives reasonable results for the four cases.

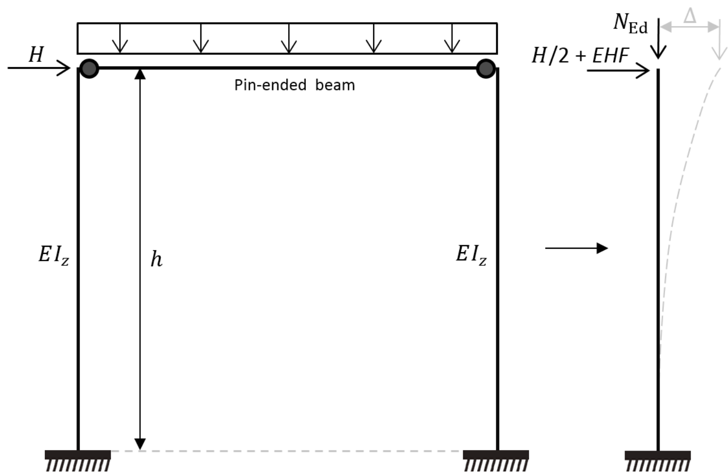
The design of the column based on **Method 2** (2.1 by a numerical *P-Δ* or 2.2 considering k_{sw}) and **Method 3** will be undertaken for the structure in Figure 3. Two examples are considered for different levels of horizontal load. A comparison of the Unity factor (UF) for relevant checks according to EN 1993-1-1 is presented in Table 2.

From Worked Example 1, it can be noted that there is very close agreement in the utilization factor between methods 2.1 and 2.2. Method 3 is conservative for $N_{Ed} = 75$ kN and $H/2 = 10$ kN. If the horizontal load $H/2$ is increased to 20 kN, Method 3 becomes unconservative.

Boundary conditions	Section	l [m]	Theoretical value of $N_{cr,z}$ [kN]	$N_{cr,1}$ 1 FE	$N_{cr,3}$ 3 FE	$N_{cr,5}$ 5 FE	$N_{cr,10}$ 10 FE
Cantilever	254 UC 107	10	$N_{cr,z} = \frac{\pi^2 EI}{(2l)^2} = 307.16$	309.47	307.19	307.17	307.16
Pinned Pinned	254 UC 107	10	$N_{cr,z} = \frac{\pi^2 EI}{l^2} = 1228.65$	1493.86	1230.59	1228.91	1228.66
Pinned Fixed	254 UC 107	10	$N_{cr,z} = \frac{\pi^2 EI}{(0.6992l)^2} = 2513.18$	3734.64	2528.93	2515.68	2513.64
Fixed Fixed	254 UC 107	10	$N_{cr,z} = \frac{\pi^2 EI}{(0.5l)^2} = 4914.59$	∞*	5022.36	4930.35	4915.65

* - See Part 1[5], Figure 8; this example represents $N_{Ed}/N_{cr} = 4$);

Table 1: Buckling analysis of a strut considering different number of finite elements (FE)[13].



$h = 10$ m;
Columns: 254 UC 107; $I_z = 5928$ cm^4; Steel: S355 JR
$N_{Ed} = 75$ kN; Example 1.1: $H = 20$ kN;
Example 1.2: $H = 40$ kN (factored loads)
EN 1993-1-1 section 5.3.2:

$$EHF = \frac{1}{200} * 75 = 0.375 \text{ kN}$$

$$N_{cr} = \frac{\pi^2 EI_z}{(2l)^2} = 307.16 \text{ kN}$$

If the system is represented by a single column:

$$\alpha_{cr} = \frac{N_{cr}}{N_{Ed}} = \frac{307.16}{75} = 4.10$$

As $\alpha_{cr} > 4$, local bow imperfections can be disregarded in the analysis – EN 1993-1-1 5.3.2 (6).

$$k_{sw} = \frac{1}{1\text{-}1/\alpha_{cr}} = \frac{1}{1\text{-}1/4.10} = 1.32$$

Figure 3: Example of a slender simple column.

Worked example	Method	l_{eff} [m]	N_{Ed} [kN]	$H/2 + EHF$ [kN]	First order bending moment [kNm]	Second order bending moment [kNm]	UF
1.1	2.1	10	75	10.375	103.75	131.29	0.53
	2.2	10	75	10.375	103.75	103.75 ∗ 1.32 = 136.95	0.55
	3	20	75	10.375	103.75	-	0.63
1.2	2.1	10	75	20.375	203.75	257.77	1.04
	2.2	10	75	20.375	203.75	203.75 ∗ 1.32 = 268.95	1.09
	3	20	75	20.375	203.75	-	0.96

Table 2: Results for two different load arrangements: simple column[13].

Worked example 2: three-storey frame

In worked example 2, the comparisons are extended to a three-storey frame (shown in Figure 4). Geometric conditions can be found in Table 3. Two examples are considered for different levels of horizontal load. Comparisons of the Unity factor (UF) for relevant checks according to EN 1993-1-1 are presented in Table 4 and Table 5 for the two horizontal load arrangements.

The effective buckling lengths were obtained by a back-calculation based on the global buckling mode of the frame. Example for Model 4:

$$N_{cr,AB} = \alpha_{cr}\, N_{Ed,AB} = \pi^2\, EI_{z,AB} / (l_{eff,AB})^2 \;\; \text{so,} \;\; l_{eff,AB} = \sqrt{\frac{\pi^2 EI_{z,AB}}{5.87 * 4055.47}} = 4.23\ \text{m}$$

Note: this process was adopted to obtain as much precision as possible in the comparison between the methods. It should be highlighted that the back-calculation method based on α_{cr} is only valid for the considered load arrangement. Conservative results for the effective lengths are expected when using approximated methods which are valid for any load arrangement.

The numerical consideration of global *P-Δ* effects and the approximate consideration of those effects with the amplification factor show a very close agreement in the utilization factor (as for worked example 1). The effective length method still gives a reasonable answer in comparison to the other two methods, but differences around 0.15 in the utilization factor (conservative or non-conservative) can be obtained.

Model	Bases	Beams	Columns	I_z [mm^4]	S [m]	h_1 [m]	h_2 [m]	h_3 [m]	$0.25N_{cr,0,AB}$ [kN]
1	Pinned	UB 457 191 161	UC 356 406 551	82670	10	3.75	3.00	3.00	30461.02
2	Pinned	UB 457 191 161	UC 356 406 340	46850	10	4.00	3.20	3.20	15172.20
3	Fixed	UB 457 191 161	UC 356 406 235	30990	10	5.00	4.00	4.00	6423.04
4	Fixed	UB 457 191 161	UC 356 368 177	20530	10	5.00	4.00	4.00	4255.08

Vertical loads on each story (unfactored): self-weight; permanent loads: 50 kN/m; imposed loads: 35 kN/m;
Horizontal loads: Example 2.1: $H = EHF$; Example 2.2: $H = EHF + 100$ kN (imposed load, unfactored) on each storey;
EHF: $\emptyset = 1/200$; Column spacing: 10 m; $h_1/h_2 = h_1/h_3 = 1.25$); Material: S355 JR;
Columns under minor axis bending; Beams under major axis bending; 10 Finite elements per member;
The solution for Model 4 was configured to achieve $N_{Ed} > 0.25\, N_{cr,0}$ (clause 5.3.2 (6) of EN 1993-1-1).

Table 3: Models considered in worked example 2.

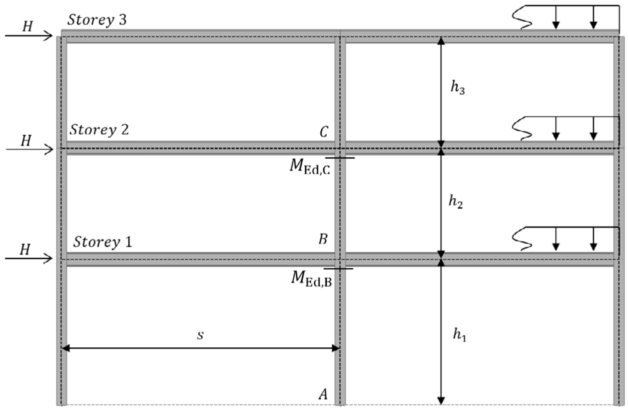


Figure 4: Geometry for worked example 2.

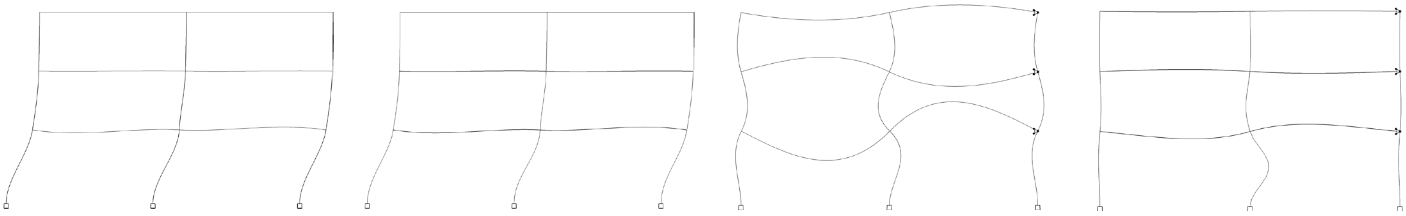
Note: in real design cases, perfectly fixed bases are not realistic. Nominally fixed bases may be assumed with the flexural stiffness of the base equal to the flexural stiffness of the column[7].

Model	α_{cr}	k_{sw}	Design method	$M_{Ed,B}$ [kNm]	$M_{Ed,C}$ [kNm]	$N_{Ed,AB}$ [kN]	$N_{Ed,BC}$ [kN]	$l_{eff,AB}$ [m]	$l_{eff,BC}$ [m]	UF_{AB}	UF_{BC}
1	7.33	1.16	2.1	60.84	33.60	3860.91	2549.80	3.75	3.00	0.21	0.13
			2.2	61.04	34.85	3860.53	2549.54	3.75	3.00	0.21	0.13
			3	52.62	30.04	3860.53	2549.54	7.78	9.58	0.30	0.24
2	4.42	1.29	2.1	70.66	33.19	3906.91	2585.93	4.00	3.20	0.36	0.21
			2.2	72.01	36.34	3906.07	2585.50	4.00	3.20	0.36	0.22
			3	55.83	28.17	3906.07	2585.51	7.50	9.22	0.49	0.39
3	8.25	1.14	2.1	35.71	26.90	3989.55	2645.51	5.00	4.00	0.52	0.32
			2.2	35.93	27.84	3988.02	2644.78	5.00	4.00	0.52	0.32
			3	31.52	24.42	3988.02	2644.78	4.42	5.43	0.49	0.36
4	5.87	1.21	2.1	38.17	26.49	4057.78	2693.40	5.00	4.00	0.64	0.40
			2.2	39.00	28.35	4055.47	2692.36	5.00	4.00	0.64	0.40
			3	32.23	23.43	4055.47	2692.37	4.23	5.19	0.66	0.49

Table 4: Worked example 2.1: horizontal loads with only EHF[13].

Model	α_{cr}	k_{sw}	Design method	$M_{Ed,B}$ [kNm]	$M_{Ed,C}$ [kNm]	$N_{Ed,AB}$ [kN]	$N_{Ed,BC}$ [kN]	$l_{eff,AB}$ [m]	$l_{eff,BC}$ [m]	UF_{AB}	UF_{BC}
1	7.31	1.16	2.1	821.33	453.69	3861.26	2549.73	3.75	3.00	0.48	0.26
			2.2	823.99	470.48	3860.86	2549.48	3.75	3.00	0.48	0.27
			3	710.34	405.58	3860.81	2549.49	7.79	9.59	0.55	0.36
2	4.41	1.29	2.1	953.98	448.26	3907.42	2585.94	4.00	3.20	0.87	0.41
			2.2	972.20	490.65	3906.37	2585.45	4.00	3.20	0.88	0.43
			3	753.64	380.35	3906.29	2585.47	7.51	9.23	0.96	0.57
3	8.23	1.14	2.1	482.07	363.31	3990.08	2645.67	5.00	4.00	0.80	0.51
			2.2	485.11	375.79	3988.33	2644.74	5.00	4.00	0.81	0.52
			3	425.53	329.64	3988.29	2644.74	4.42	5.43	0.75	0.54
4	5.86	1.21	2.1	515.31	357.77	4058.47	2693.69	5.00	4.00	1.09	0.69
			2.2	526.53	383.70	4055.75	2692.33	5.00	4.00	1.14	0.71
			3	435.14	316.28	4055.70	2692.34	4.23	5.19	0.98	0.71

Table 5: Worked example 2.2: horizontal loads with EHF + 100 kN (imposed load, unfactored)[13].



a) Sway frame: 1 Finite element per member: $\alpha_{cr} = 5.94$

a) Sway frame: 10 Finite elements per member: $\alpha_{cr} = 5.87$

c) Non-sway frame: 1 Finite element per member: $\alpha_{cr} = 41.63$

d) Non-sway frame: 10 Finite elements per member: $\alpha_{cr} = 14.81$

Figure 5: Influence of the number of finite elements per member on frame stability[13]***.***

Influence of the number of finite elements on frame stability:

The differences in modelling precision are demonstrated in Figure 5, which shows the different buckling modes and values of α_{cr} for models with 1 and 10 finite elements per member (using Model 4 from worked example 2.1). The non-sway frame has horizontal supports on each floor level.

Calculation of α_{cr} using the Horne method:

For model 4 of worked example 2.1, the calculation of α_{cr} according to clause Section 5.2 of EN 1993-1-1 is shown in Figure 6. The approximate value of 6.61 may be compared with the precise value of 5.87 from Table 4 and 5.86 from Table 5. The approximated value of 6.61 is the same for worked examples 2.1 and 2.2, as the ratio $H_{Ed}/\delta_{H,Ed}$ is identical in the method.

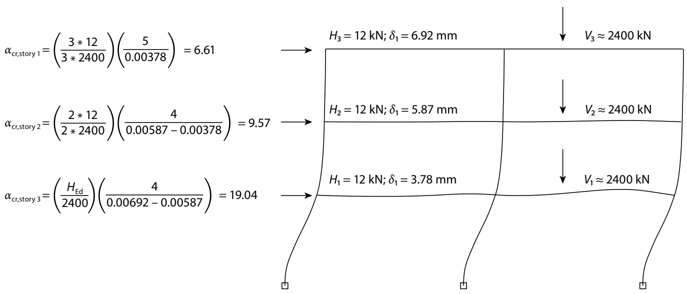


Figure 6: Calculation of α_{cr} with the Horne method (worked example 2.1)[13]***.***

Conclusions

1. Eurocode 3 provides essentially 3 different methods to consider local and global second order effects when verifying members;
2. In practice, local second order effects are usually considered when checking member stability according to section 6.3 of EN 1993-1-1;
3. Local imperfections may need to be considered for global analysis; this may be mandatory according to clause 5.3.2 (6) of EN 1993-1-1; the criteria is more significant for frames with fixed bases where lower α_{cr} can be obtained with slender members;
4. The effective length method considers the effects of global second order effects by increasing the local second order effects; buckling lengths greater than $2l$ may be required;
5. The numerical consideration of global *P-Δ* effects and the approximated consideration of those effects with the amplification factor give very similar results; For member stability verifications according to section 6.3 of EN 1993-1-1, system lengths should be used;
6. The effective length method gives a reasonable answer in comparison to the other two other methods where second order internal forces are calculated. Differences between methods can be up to approximately 0.15 in the utilization factor (conservative or non-conservative); differences are less significant for higher values of α_{cr}.
7. The importance of considering more than 1 finite element per member was demonstrated for struts and frames. At least 3 finite elements are recommended;
8. Horizontal loads have a small influence in the values of α_{cr}.

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AD 428: Slip factors for alkali-zinc silicate paint

This AD note draws attention to the slip factors for alkali-zinc silicate painted faying surfaces considered in AD 383 which have been updated in the 2018 revision of BS EN 1090-2.

AD 383, which was published in September 2014, discussed the slip factor for surfaces coated with alkali-zinc silicate paint and the significant influence of the coating thickness. The AD referred to forthcoming changes to Table 18 of BS EN 1090-2, expected to reflect concerns about the relationship between the coating thickness and slip factor. In the interim, AD 383 proposed slip factors of 0.3 (if certain recommended practices were followed) or 0.2 as a conservative value.

BS EN 1090-2 was revised in 2018 and slip factors are presented in Table 17. For surfaces coated with alkali-zinc silicate paint, the nominal thickness is now specified as 60 µm, with a dry film thickness between 40 µm and 80 µm.

If the applied coating meets the thickness limits specified in Table 17, a slip factor of 0.4 may be assumed. AD 383 noted that in practice the coating thickness can often exceed 80 µm, so coating procedures will need to be carefully controlled and the dry film thickness measured, to ensure the limits in Table 17 are satisfied. If such control is not practical, then the conservative slip factors quoted in AD 383 may be adopted.

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