Introduction to fatigue design to BS EN 1993-1-9

The assessment of fatigue performance is routine in bridge design but is only relevant to specific elements in buildings which may suffer from fatigue damage. One example of these is crane runway beams. Richard Henderson of the SCI introduces some of the background.

Introduction

The phenomenon of metal fatigue involves the development of cracks in elements that are subject to many repeated applications of loads which are lower than the maximum loads to which the element is subjected. If fatigue cracks develop unnoticed, they will eventually result in complete failure of the element with potentially catastrophic consequences.

History

Research into fatigue in metal structures began as early as 1837 with tests on conveyor chains. A locomotive axle failure due to fatigue was recognized as the cause of a train accident at Meudon, near Versailles in 1842. F Braithwaite coined the term fatigue in his report "On the fatigue and consequent fracture of metals" published in the ICE minutes of proceedings in 1854. August Wohler conducted systematic investigations into metal fatigue of railway axles over a 20 year period from 1852, produced S-N curves illustrating fatigue behaviour and introduced the idea of an endurance limit. In 1945, A M Miner developed a design tool based on the Palmgren linear damage hypothesis. The stress raising effect of small-radius corners and the consequent effect on fatigue behaviour was established following investigation into the Comet air disasters of 1953 and 1954.

Basic Concepts

Fatigue cracks usually initiate at a surface defect such as a sharp corner or a weld toe and develop when subject to fluctuating stresses above a certain threshold level. The endurance of a detail or component is the number of cycles to failure under a fluctuating stress of a constant amplitude. A point can be plotted on a graph with the number of cycles to failure (N) as abscissa and the constant amplitude stress (S) as ordinate. Stress range is defined as the algebraic difference between the two extremes of a stress cycle so the constant amplitude fluctuating stress is a constant stress range. By plotting the endurance for each constant stress range, a curve called an S-N curve can be drawn, the typical form of which is shown in Figure 1 on a semi-log plot.

The S-N curve exhibits a negative gradient such that a longer endurance corresponds to a lower stress range. Stresses below a stress range magnitude called the cut-off limit do not cause fatigue damage. According to Miner's rule, fatigue damage can be summed linearly for a given detail using the S-N curve to determine the number of cycles to failure N_i for stress range $\Delta\sigma_i$. If the detail is subject to a number of cycles n_i for the corresponding stress range, the fatigue damage can summed for k stress ranges and must be no greater than 1.0. The relevant expression is:

$$\sum_{i=1}^{k} \frac{n_1}{N_1} \le 1.0$$

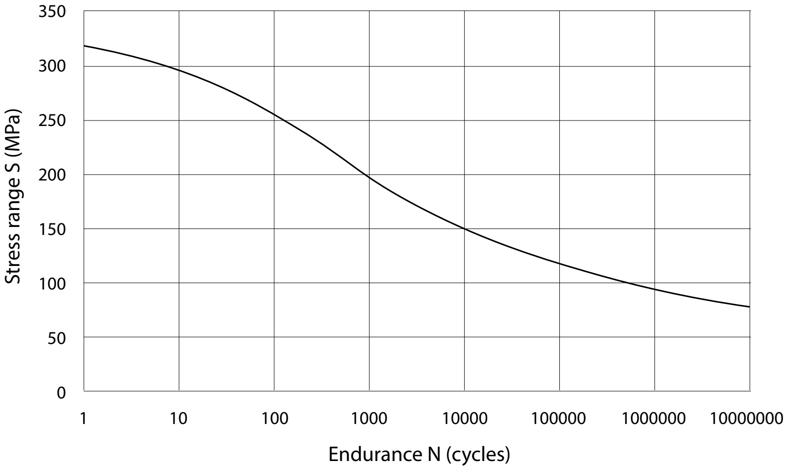

Figure 1: Example S-N Curve

Defects in plain steel, welded joints and welded attachments all affect the fatigue life of a detail. As a result, many fatigue tests have been carried out on different details to develop S-N curves that can be used for fatigue damage calculations. Details are tabulated in BS EN 1993-1-9 (hereinafter denoted EC3-1-9) and are separated into the following headings.

Table No.	Heading
8.1	Plain members and mechanically fastened joints
8.2	Welded built-up sections
8.3	Transverse butt welds
8.4	Weld attachments and stiffeners
8.5	Load carrying welded joints
8.6	Hollow sections (t ≤ 12.5 mm)
8.7	Lattice girder node joints
8.8	Orthotropic decks – closed stringers
8.9	Orthotropic decks – open stringers
8.10	Top flange to web junction of runway beams

Within each table, details are identified and provided with an identifying number which corresponds to the relevant S-N curve.

The S-N curves for various classes of detail have been idealized in EC3-1-9 into a set of parallel lines with straight segments, plotted on a logarithmic scale on both axes and those for direct stress are shown in Figure 7.1 of the standard. The S-N curves are identified by a detail category number $\Delta\sigma_c$ which corresponds to the reference fatigue strength in MPa for the detail which is equal to the constant amplitude stress range for an endurance of 2 × 10^6 cycles. The curves are shown in Figure 2.

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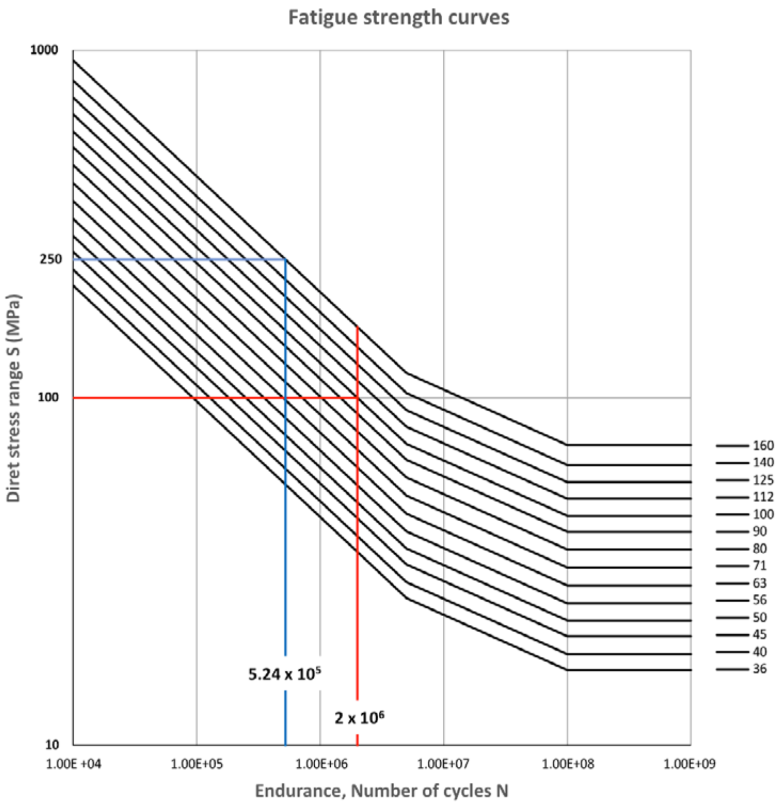

Figure 2: Fatigue strength curves for direct stress ranges

▶29 The equation for the sloping part of the curves is of the form:

$$\Delta\sigma_R{}^m N_R = \Delta\sigma_C{}^m 2 \times 10^6$$

with m = 3 for $N \leq 5 \times 10^6$ and:

$$\Delta\sigma_R{}^m N_R = \Delta\sigma_C{}^m 5 \times 10^6$$

with m = 5 for $5 \times 10^6 \leq N \leq 10^8$.

The first equation can be expressed as:

$$3 \times \log_{10}\Delta\sigma_R + \log_{10}N_R = 3 \times \log_{10}\Delta\sigma_C + \log_{10}2 \times 10^6$$

This is a straight line on a log-log plot with gradient -1/3. As an example of their use, for detail category 160 (plates and flats with as-rolled edges, with sharp edges, surface and rolling flaws removed by grinding until a smooth transition is achieved; $\Delta\sigma_C$ = 160 MPa – see Table 8.1 of EC3 1 9), the endurance for a nominal direct stress range of 250 MPa is given by:

$$3 \times \log_{10}250 + \log_{10}N_R = 3 \times \log_{10}160 + \log_{10}2 \times 10^6$$

$$N_R = 5.243 \times 10^5$$

ie the endurance at a constant amplitude stress range of 250 MPa is about 524,000 cycles.

Fatigue loading

Fatigue loading usually involves a spectrum of loads of different magnitudes. A spectrum can be built up for a particular structural action which can then be converted into a stress history.
A method for determining the magnitude of stress ranges from a stress history is known as the reservoir counting method and is described in Published Document PD 6695-1-9:2008.
The reservoir counting method is illustrated in Figure 3.

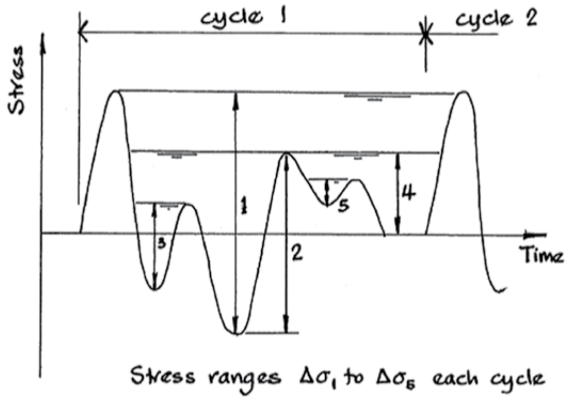

Figure 3: Reservoir counting method

The load spectrum may be continuous (such as for wave loading) and be describable by fitting a probability distribution to measured data. The data can then be discretized and a histogram of the number of loads of different magnitudes produced. The stress ranges corresponding to each load magnitude can then be determined.

Fatigue Assessment and Verification

Two methods of fatigue assessment are described in EC3-1-9: the safe life method and the damage tolerant method. The safe life method of assessment is considered in what follows. For some circumstances, a simple method of fatigue assessment can be used which does not refer to a load spectrum. The method is set out in EC3-1-9 and involves verification in the stress domain; it is described below.

Sections 5 and 6 of the standard provide details of how to calculate the stresses for assessing the fatigue performance of a detail. Nominal values of stresses should be calculated at the serviceability limit state according to elastic theory, excluding stress concentration effects. The nominal direct and shear stresses should be calculated at the site of potential initiation of a fatigue crack. The nominal stresses are modified by a stress concentration factor if the relevant nominal stress is affected by a local geometric feature, such as an opening with radiused corners. Stress concentration factors are provided in Figure 4 of PD6695-1-9:2008. Stresses in welds are calculated using a different formula from that given in BS EN 1993-1-8 for weld design, as indicated in Section 5(6). For certain details shown in Table B.1 of EC3-1-9, fatigue resistance can be determined using the geometrical (hot spot) stress method. Stress ranges for fatigue design are based on nominal stresses, modified nominal stresses or geometrical (hot spot) stress ranges.

For the structure and loading under consideration, the relevant part of EN 1993 may provide parameters for calculating the design value of the nominal stress ranges for fatigue verification. Using this approach, the design value of the nominal, modified nominal or geometrical stress range factored for fatigue must be less than the reference fatigue strength at 2 million cycles for each detail identified in tables 8.1 to 8.10.

The design value of nominal stress ranges is given in Section 6.2 of EC3-1-9 as

$$\gamma_{\mathrm{Ff}}\,\Delta\sigma_{\mathrm{E,2}} = \lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4 \ldots \times \lambda_n \times \Delta\sigma(\gamma_{\mathrm{Ff}}\,Q_k)$$

for direct stresses where $\Delta\sigma(\gamma_{\mathrm{Ff}}\,Q_k)$ is the stress range caused by the fatigue loads specified in EN 1991 and the λ_i are damage equivalent factors depending on the spectra in the relevant parts of EN 1993. The product of the damage equivalent factors λ_i adjusts the stress ranges caused by the fatigue loads into stress ranges corresponding to 2×10^6 cycles.

The fatigue verification involves checking that the nominal, modified nominal or geometrical stress ranges due to frequent loads $\Psi_1 Q_k$ do not exceed the following limits:

$$\Delta\sigma \le 1.5 f_y \quad \text{for direct stress ranges}$$

$$\Delta\tau \le (1.5 f_y)/\sqrt{3} \quad \text{for shear stress ranges}$$

Under fatigue loading, the following two inequalities should be verified:

$$\frac{\gamma_{\mathrm{Ff}}\,\Delta\sigma_{\mathrm{E,2}}}{\Delta\sigma_{\mathrm{C}}/\gamma_{\mathrm{Mf}}} \le 1.0$$

$$\frac{\gamma_{\mathrm{Ff}}\,\Delta\sigma_{\mathrm{E,2}}}{\Delta\tau_{\mathrm{C}}/\gamma_{\mathrm{Mf}}} \le 1.0$$

The design value of the nominal stress ranges should therefore be less than the reference fatigue strength at 2 million cycles for that particular detail.

In addition, for stress ranges of combined shear and direct stress a further inequality should be satisfied:

$$\left(\frac{\gamma_{\mathrm{Ff}}\,\Delta\sigma_{\mathrm{E,2}}}{\Delta\sigma_{\mathrm{C}}/\gamma_{\mathrm{Mf}}}\right)^3 + \left(\frac{\gamma_{\mathrm{Ff}}\,\Delta\sigma_{\mathrm{E,2}}}{\Delta\tau_{\mathrm{C}}/\gamma_{\mathrm{Mf}}}\right)^5 \le 1.0$$

Lambda values which allow this approach are given in BS EN 1991-3 for cranes and in BS EN 1993-2 for bridges.

UK National Annex

The UK National Annex to EC3-1-9 states that where no λ_i values are given the relevant parts of EC3, the verification should be based on the damage accumulation equation which is essentially the equation for Miner's rule:

$$D_d = \sum_{i=1}^{n} \frac{n_{\mathrm{Ei}}}{N_{\mathrm{Ri}}} \le 1.0$$

The most comprehensive load model available should be used to establish a spectrum of stress ranges. The spectrum consists of a series of bands of stress $\Delta\sigma_i$ which should be multiplied by the load factor γ_{Ff}. The reference fatigue strength values $\Delta\sigma_{\mathrm{C}}$ divided by γ_{Mf} are used to obtain the endurance value N_{Ri} for each band.

In the equation for damage, n_{Ei} is the number of cycles associated with the stress range $\gamma_{\mathrm{Ff}}\,\Delta\sigma_i$ for band i in the factored spectrum and N_{Ri} is the endurance in cycles obtained from the factored $\frac{\Delta\sigma_{\mathrm{C}}}{\gamma_{\mathrm{Mf}}} - N_{\mathrm{R}}$ curve for a stress range of $\gamma_{\mathrm{Ff}}\,\Delta\sigma_i$.

It is intended to give a more detailed discussion of a fatigue check in an example in a subsequent article.