The resistance of cross sections subject to shear and bending – theoretical analysis and practical design rules

Sections subject to both bending and shear have a reduced bending resistance where the shear force is greater than half the shear resistance. Richard Henderson of the SCI discusses the background and design rules.

Work carried out between 1930 and 1965 on the resistance of cross sections capable of being designed plastically was presented by Baker, Horne and Heyman[1]. Theoretical treatments of the effect of shear force on the resistance moment of sections were developed and were subsequently compared with tests. The design rules presented in BS 5950-1:2000 and subsequently in BS EN 1993-1-1 were based on this work.

Horne[2] examined rectangular and I sections and developed expressions for the reduction in the bending resistance of cross sections where the sections are subject to both bending and shear. In the examination, the sections are assumed to be capable of carrying their full plastic moment: sections are assumed to be restrained from global buckling and I sections are either class 1 or class 2 according to EC3.

Rectangular Section

A rectangular section will carry a bending moment equal to its elastic moment of resistance where only the extreme fibres reach yield stress. The remainder of the cross section is able to resist a shear force. The shear stress distribution is parabolic over the depth of the section and is zero at the extreme fibres with a maximum value at the neutral axis. The average shear stress is two thirds of the maximum value. If the bending moment is increased above the elastic moment of resistance, the area of the section available to resist shear is reduced until it vanishes when the plastic moment of resistance is reached. At this point, the whole section reaches its yield stress. The plastic resistance moment of the section is $M_p = (bh^2/4)f_y$ and its plastic shear resistance is $V_y = bh\tau_y$ if the bending and shear are each considered on their own.

When the bending moment is between the elastic and plastic moment of resistance, the elastic core of the section has a depth y_o above and below the neutral axis and $y_o < h/2$ where h is the depth of the section. The resistance moment is given by the sum of the plastic moment of resistance of the outer portion and the elastic moment of resistance of the core:

$$M = b/4(h^2 - 4y_o^2)\sigma_y + 2/3by_o^2\sigma_y$$

and the shear resistance is provided by the core and given by $V = 4/3by_o\tau_y$.

Eliminating y_0 and using the expressions for M_p and V_p gives:

$$M_{pr}/M_p = 1 - 3/4(V/V_p)^2 \qquad (1)$$

M_{pr} is the reduced plastic moment of resistance in the presence of shear. The expression is valid for values of V up to that for which $y_o = h/2$ ie $V/V_p \leq 2/3$.

Horne showed that using the Tresca yield criterion, a less conservative estimate is given by $M_{pr}/M_p = 1 - 0.444(V/V_p)^2$ provided $V/V_p \leq 0.792$.

The interaction between shear and bending according to this expression is shown in Figure 1

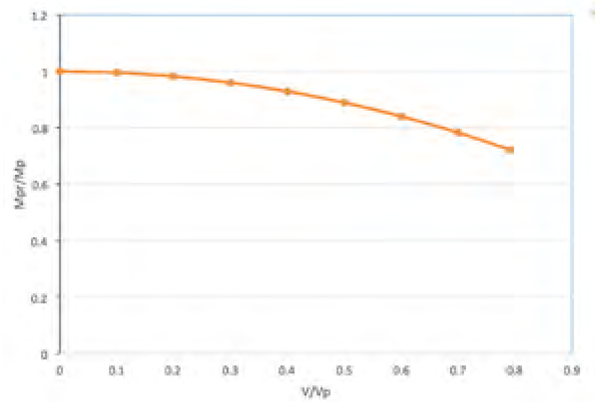

Figure 1: Interaction of shear and bending – Rectangular section

According to the less conservative estimate, the bending resistance of the section is about 89% of the plastic resistance moment when the shear force is half the shear resistance.

I Section

A similar analysis can be made of an I section, if the shear stresses are assumed only to be in the web. The plastic resistance moment of the web is denoted by $M_{pw} = (d_w^2t_w/4)\sigma_y$ and the shear resistance by $V_{pw} = d_wt_w\tau_w$, where d_w and t_w are the depth and thickness of the web. Using equation 1, the reduced plastic moment is given by:

$$M_{pr} = M_p - 3/4(V/V_{pw})^2 M_{pw}.$$

This equation is valid provided $V/V_{pw} \leq 2/3$ which means that the plastic zones in the section extend beyond the flanges and into the web.

Horne and Morris[3] discussed the effect of shear force on the plastic moment, assuming the web of the I section provides all the shear resistance and the shear stress τ_w is assumed to be uniform over the depth of the web. The longitudinal bending stress in the web is reduced because of the presence of the shear stress to a value which can be determined using the Von Mises yield criterion: $\sigma_w = [f_y^2 - 3\tau_w^2]^{0.5}$. The reduction in longitudinal bending stress in the web results in a reduced bending resistance given by:

$$M_{pr} = M_p - M_{pw}[1 - \{1 - (V/V_{pw})^2\}^{0.5}]$$

The interaction between the bending moments and the ratio of the applied shear force and shear resistance is shown in Figure 2.

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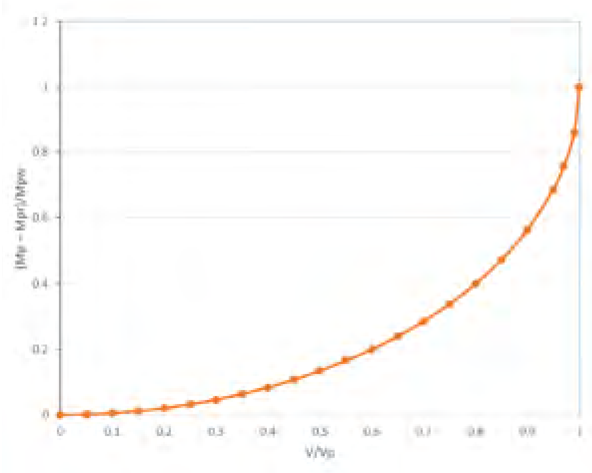

Figure 2 Effect of shear force on plastic moment of resistance of an I section

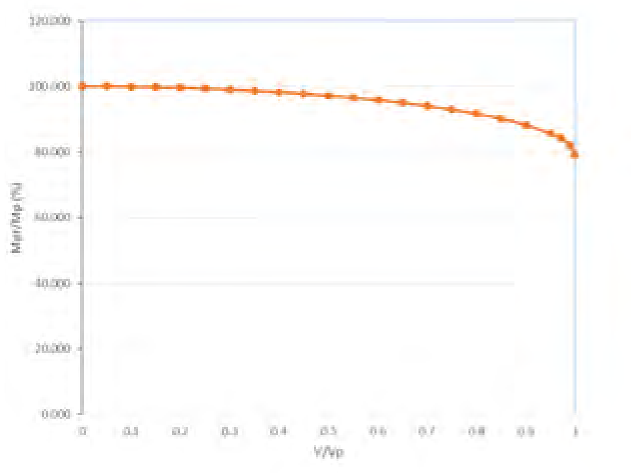

Figure 3 Reduction in plastic resistance moment for increasing ratio of shear force to shear resistance

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The value of $(M_p - M_{pr})/M_{pw}$ where the shear force is half the shear resistance of the web is 0.134. The reduction in plastic bending resistance of the section is therefore about 13% of the plastic bending resistance of the web. For a 400 mm deep I section with 180 mm wide flanges 15 mm thick and an 8 mm thick web, the reduction in the full plastic bending resistance is only 3% under a shear force of half the shear resistance of the web. Figure 3 shows the relationship between plastic resistance moment and the ratio of shear force to shear resistance of the web for the I section discussed.

If bending about the minor axis of an I section is considered the behaviour is similar to a rectangular section and the shear stress is distributed parabolically over the width of the flanges and the bending stress distribution is also non-linear. The reduced bending resistance is given by Horne and Morris as:

$$M_{pr} = M_p[1 - 0.45(\tau_w/\tau_y)^2]$$

where τ_w is the shear stress calculated on the area of the flanges. If the shear force on the section is half the shear resistance of the flanges then the reduced resistance moment is about 89% of the full plastic resistance moment ie as found earlier.

Results of tests and design rules

Despite the foregoing analysis, the results of tests and also of advanced theory shows that there is no reduction in the resistance moment due to the presence of shear unless the shear force approaches the shear resistance of the section. This is because the portions of a beam section which are subject to both high shear and high bending stresses are limited in extent and are surrounded by elastic zones so plastic flow is largely prevented. The locations in a structure where both bending and shear may be significant are limited: the root of a cantilever and at the central support of a two–span beam are two possible locations.

The design rules in BS 5950-1:2000 and BS EN 1993-1-1 adopt a safe approach to the effect of shear force on the resistance moment and allow the full plastic resistance moment to be used in conjunction with a shear force of up to half the shear resistance of a beam. In fact BS 5950 was slightly more generous than EC3 and no reduction in bending resistance was required for shear force up to 60% of the shear resistance. The contribution of the shear area of the section to the bending resistance is reduced when the shear force on the section exceeds half the shear resistance. Figure 4 shows the percentage reduction in resistance

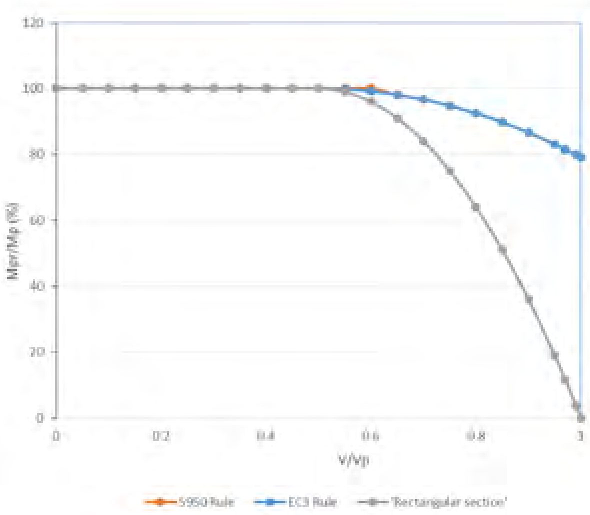

Figure 4 Reduction in resistance moment due to shear

moment according to both EC3 and BS 5950 for the 400 mm deep beam. The difference in the treatment is insignificant.

The reduction in minor axis bending resistance when the section is subject to a shear force is also shown in Figure 4, labelled Rectangular Section. Unlike the I section, the bending resistance reduces significantly under high shear and reduces to zero when the shear force reaches the shear resistance because the maximum shear stress of $f_y/\sqrt{3}$ is present over the full extent of the flanges. This effect also applies to rectangular sections. For a Tee section, the stem of the Tee provides the shear resistance but also develops longitudinal stresses to provide the bending resistance. These stresses are reduced in the presence of shear in a similar way to those in a rectangular section.

References

1 Sir John Baker, M R Horne and J Heyman, The Steel Skeleton, Volume Two, Plastic behaviour & design, 1956, Cambridge University Press

2 M R Horne, Plastic theory of structures, 1979, Pergamon Press

3 M R Horne and L J Morris, Plastic design of low resistance rise frames, 1981, Granada Publishing

AD 417: Resistance of sections to combined shear and bending

This Advisory Desk note reminds designers that the form of the section has a significant impact on the reduction of bending resistance under high shear.

Clause 6.2.8 of BS EN 1993-1-1:2005 deals with the resistance of cross sections to combined bending and shear and first of all states:

(1) Where the shear force is present allowance should be made for its effect on the moment resistance.

It then goes on to say:

(2) Where the shear force is less than half the plastic shear resistance its effect on the moment resistance may be neglected except where shear buckling reduces the section resistance, see EN 1993-1-5.

(3) Otherwise the reduced moment resistance should be taken as the design resistance of the cross-section, calculated using a reduced yield strength … for the shear area.

The reduced yield strength depends on the ratio of design shear force to the shear resistance of the section.

For an I section, the shear area approximates to the area of the web and the flanges still provide their full resistance moment so the reduction in bending resistance may not be more than about 20% when the design shear force equals the shear resistance. For a rectangular section, the full section forms the shear area so the bending resistance reduces to zero under the same circumstances. A Tee section would also behave in a similar way.

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