

A view of Torsion – Part Three

Parts One and Two introduced the two torsion resistance mechanisms available to a steel I-section and described them separately. In this final Part of the series, Alastair Hughes discusses the interaction of the two mechanisms.

Introduction

To begin with, a reminder:

- Forget about ‘warping’ unless the member possesses two flanges
- The option exists to ignore St Venant and resist torsion by ‘warping’ alone

What follows is for designers who need to extract all available stiffness and/or resistance from a conventional I- or H-beam by taking advantage of both ‘warping’ and St Venant. It does not apply to shapes like angles and tees, whose warping is insignificant, nor to hollow sections, whose warping is either non-existent (in the case of CHS) or disregarded. Angles, tees and hollow sections are designed to resist torsion by St Venant alone.

As reassurance, compare the tabulated properties of 178 × 203 × 37 UKT and the 406 × 178 × 74 UKB split to make it. Whereas I_t is just under half that of the parent section (as the membrane analogy would indicate), I_w is less than a thousandth (and hardly seems to warrant a column in the table).

The problem

While each resistance mechanism is comprehensible enough on its own, and both can deliver resistance independently of one another, the proportions in which they share the burden will vary along the length of the member in a manner which is obscure to those of us not qualified in higher mathematics.

Although EN 1993-1-1 6.2.7(2) might give the impression that there are two kinds of torsional moment, there is only one – but there are two kinds of resistance, and equilibrium demands that the sum of their respective contributions must equal the torsional moment T_E at any and every point along the member. Of course T_E itself may also vary lengthwise, which adds to the complexity.

The mechanism known as warping is actually differential flange bending. Like any other beam, the flange is much more efficient when its span/depth ratio is small. Double the span and the deflection will multiply by eight. By contrast, St Venant displacement merely increases pro rata with the length. Naturally, therefore, ‘warping’ can be expected to provide most of the stiffness and resistance if the member is short, and St Venant will assume dominance as the length increases. In between, both have a helpful contribution to make – and their combined effect can improve on the sum of their independent efforts. Many practical beams occupy this ‘in between’ zone.

Formulating the problem in general terms is not too difficult. Here it is on a stamp, (top of next column) commemorating the great Ukrainian engineer S P Timoshenko, who is the hero of this episode in the story of torsion.

The differential equation on the stamp is essentially what would, in current symbolism, be expressed as:

$$T_E = G I_t \varphi' - E I_w \varphi'''$$



In words, the St Venant contribution $G I_t \varphi'$ and the ‘warping’ contribution $E I_w \varphi'''$ add up to the torsional moment. (Don’t be concerned by the minus sign, which is as in the familiar bending formula $M_y = -E I_z''$.)

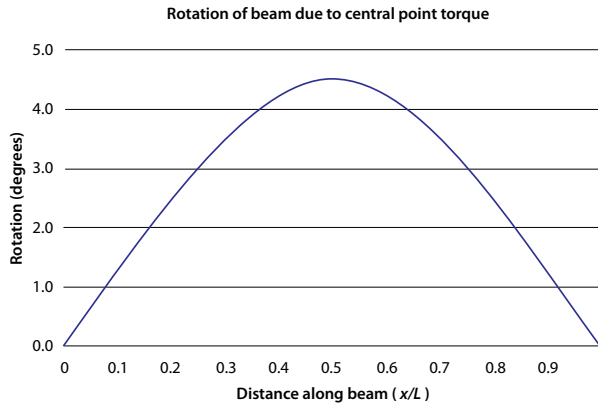
I_w , the warping constant, was formerly symbolized H . The new symbol is rather regrettable, because this section property (whose units are m^6) has even less to do with inertia than I_t [which is, for a circular bar (only), numerically the same as its polar moment of inertia]. The ‘warping’ contribution is equal to the shear force in the flange (the rate of change of its ‘warping’ moment) times the distance between flanges, taken as $(h - t_f)$. The ‘warping’ moment is $\pm E I_t y''$ and $y = \varphi (h - t_f)/2$, whence $I_w = I_t (h - t_f)^2/2$. I_t is the second moment of area of one flange, $b t_f^3/12$, and is not very different from $I_z/2$ for the section as a whole.

The mathematicians need to be employed because the equation on the stamp must simultaneously be satisfied from end to end of the beam, taking account of any lengthwise variation of T_E , whose distribution will, in general, depend not only on the torque(s) applied but also on the two kinds of torsional stiffness in play.

The solutions

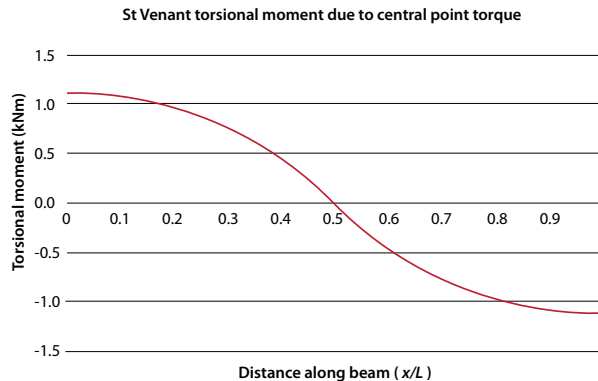
Suffice it to say that the mathematical profession has provided us with a set of solutions for a range of situations encountered in practice, and these are given in the new SCI Design Guide, P385. In P057 (the earlier publication), many of them were presented as graphs, because the formulae are heavy duty, with abundant hyperbolic functions. Twenty years on, a £7 pocket calculator will take these in its stride, so the number of graphs is somewhat reduced in P385. Some graphs are retained, however, not least because they give a visual indication of the interplay between the resistance mechanisms.

Consider, as a simple numerical example, a beam subject to point torque at midspan. Here are plots of the variation of φ and its first, second and third derivatives with respect to x . The torque T is 3.4 kNm, the span is 3.46 m and the beam is 305 × 127 × 42 UKB, S275. For this section $I_t = 21.1E-8 m^4$, $I_w = 0.0846E-6 m^6$ and $(h - t_f)/2 = 0.148 m$.



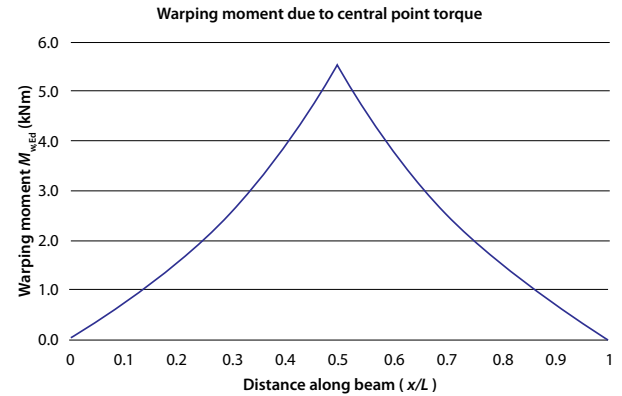
Graph 1: Beam rotation

The lateral deflection of the top flange is $\varphi(h - t_f)/2$, so this graph can be viewed as its deflected shape. At midspan, φ is 0.078 rad (4.5°) and deflection is 11.5 mm. This might well be judged excessive, even if T incorporates a partial factor of 1.5.



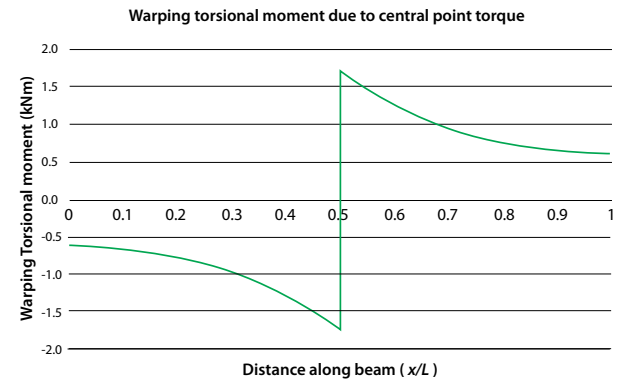
Graph 2: St Venant torsional moment

The St Venant contribution is $Gt\varphi'$, so this graph reflects its lengthwise variation. At maximum, at the ends of the beam, $\varphi' = 0.064 \text{ m}^{-1}$. The peak St Venant shear stress ($Gt\varphi'$) is 62 MPa at the flange face; 41 MPa at the web face.



Graph 3: Warping moment

'Warping' moment M_w is $\pm EI_w \varphi''$, so this graph takes the shape of the flange bending moment diagram – the warping moment diagram. At midspan M_w peaks at $\pm 5.6 \text{ kNm}$, a significant proportion of the flange's bending resistance $M_{pl,z,Rd}$ (which is 12.8 kNm).



Graph 4: Warping torsional moment

The 'warping' contribution to torsional moment is $-EI_w \varphi'''$, so this graph reflects its lengthwise variation (which mirrors graph 2). Either side of midspan, the peak warping shear force in the flange is less than 2 kN, corresponding to a peak elastic shear stress at its

neutral axis of less than 2 MPa. The shear stresses associated with warping are, in themselves, really quite trivial.

Member verification

Thanks to elastic theory and higher mathematics, the designer can now evaluate all torsionally induced stresses everywhere along and around the beam. But some head-scratching remains.

Serviceability should normally take priority, with strength checked after deformations have been judged acceptable. EN 1993-1-1 6.2.7 hints at this with its wording: 'For members... for which distortional deformations may be disregarded...' which might almost be taken to imply that if deformations **do** command regard there is no need to verify resistance. In a purely torsional design situation, that might not be far from the truth. But most practical beams have to be verified for a combination of bending and torsion.

Elastic verification using the Von Mises yield criterion is one possibility. Identifying each and every potential critical point on the beam is easier said than done, especially when stresses due to regular bending and shear have to be superposed.

Commonly, unacceptable torsional deformation of a beam will precede yield by a comfortable margin. But there are exceptions; one is a very short member in which 'warping' is dominant. Another is a beam which is subject to a large amount of regular bending plus a small amount of torsion. In a case like this, the merest whiff of torsion could 'fail' the (Class 1 or 2) beam if its presence made elastic verification compulsory.

The Eurocode is an advance on its predecessor because it does permit plastic verification of the cross-section in the normal way. This involves downgrading the yield strength if the shear force exceeds half the plastic shear resistance. The latter is subject to reduction when torsional shear stress is present (in the web, presumably) but even so it is only rarely that the yield strength, and hence bending resistance, of an I-section will have to be downgraded. In the flanges, shear stresses induced by torsion do not have the same significance.

In practice, for an I-section whose yield strength is not downgraded, it is only the flange bending moment due to 'warping' that interacts with regular bending moment. However

it is often the case that an eccentrically applied gravity load (which is responsible for the torsion) continues to act vertically while the beam rotates at its point of attachment. This induces a weak-direction moment M_z (equal to ϕM_y) as a secondary effect of torsion. So two regular moments interact with two opposing warping moments. The regular biaxial bending formula, which takes no heed of warping moment, needs to be extended, and P385 includes NCCI to this effect.

Torsion-resisting beams habitually lack restraint against lateral-torsional buckling (LTB). In this event the compression flange is liable to fail prematurely, and EN 1993-1-1 offers a choice of complicated formulae to apply, none of which takes any account of torsion. Fortunately, Professor Lindner at the Technical University of Berlin has researched this interaction, and his formula has official status as UKNA-endorsed informative Annex A to EN 1993-6, the Part dealing with crane beams. P385 adopts this formula, not just for crane beams.

In Conclusion

This series of articles has aimed to whet the reader's appetite for SCI's P385: 'Design of Steel Beams in torsion', the long-awaited revision of P057. The emphasis has been on textbook material that Standards and SCI Design Guides take as read, but some of the changes have been previewed. The new publication covers the whole subject in far greater detail, with chapters on PFC and ASB sections (which are complicated, because the axis they twist about is not their centroidal axis) and on connection design. Also included are section property tables, warping/St Venant interaction graphs and formulae, advice on serviceability and a set of design examples.

As an afterthought, reflect that it is rare for torsion to be properly modelled in skeletal analysis. With each member represented by a single line, only the St Venant stiffness can be input. 'Warping' stiffness is there in reality but not in the model, and any incorrect distribution of stiffness is liable to distort the result of an apparently precise elastic analysis. This could be to the detriment of some members of the framework. Perhaps the poor torsional performance of conventional steel sections should be viewed less as a disadvantage, more as a saving grace.