# Choice of lateral-torsional buckling curves – according to Eurocode 3 and the UK National Annex

*Alastair Hughes of the SCI explains some of the subtleties of the interaction between the UK National Annex and the Eurocode itself.*

### Introduction

Lateral-torsional buckling (LTB) is the instability phenomenon which is liable to limit the bending resistance of a beam whose compression flange is unrestrained, intermittently restrained or flexibly restrained. What distinguishes it from regular strut buckling is that the tension flange, which naturally tends to remain straight, exerts some control over the wayward compression flange to which it is attached. If the compression flange is to buckle laterally, and the tension flange is not, the beam is forced to twist – hence the description 'lateral-torsional'. All conventional sections and even some exceptionally slim and slender RHS are prone to LTB.

Eurocode 3, like BS5950, provides the designer with buckling curves from which a 'buckling resistance moment' can be derived. In Eurocode terms this is $M_{b,Rd}$ – b for buckling, R for resistance, d for design. Needless to say it is a major axis moment $M_y$. $M_{b,Rd}$ is the product of what the section's bending resistance would otherwise be, eg. $W_{pl} f_y$ for a class 1 or 2 section, and $\chi_{LT}$, a reduction factor for LTB, which is the ordinate of the buckling curve. As with struts, the buckling curves plot (or, in BS 5950, tabulate) strength reduction against slenderness, but an important difference from BS 5950 is that the slenderness in Eurocode buckling curves is 'non-dimensional'. This is tautology, since all slendernesses are non-dimensional, but it is the term that Eurocode 3 has chosen. Eurocode 4 chose 'relative slenderness' and other authors have variously chosen 'reduced' or 'generalized' slenderness. What it actually means, in the LTB context, is given by the expression:

Non-dimensional slenderness (NDS) $\bar{\lambda}_{LT} = \sqrt{\dfrac{W_y f_y}{M_{cr}}}$

where $W_y f_y$ is, again, what the bending resistance would otherwise be, depending on the class of the section, and $M_{cr}$ is what the buckling resistance would theoretically be if strength were limitless (but *E*-value same as steel). The Code itself is silent on the calculation of $M_{cr}$, regarding it as a matter for textbooks or NCCI (non-conflicting complementary information). Plenty of the latter is available and the subject has been covered in previous issues of NSC.

The advantage of the non-dimensional presentation is that separate curves are not needed for S275 and S355, nor for the stepwise reductions in yield strength at element thicknesses of (in UK) 16 and 40mm.

This article is concerned with the choice of curve for LTB once the NDS has been established.

### What buckling curves allow for

Just as with struts, beams failing by LTB deform until the weak-direction moment acting on the flange, magnified by eccentricity, exceeds available resistance. There is interaction between stiffness and strength, because as soon as the extreme fibre yields the bending stiffness of the flange is eroded. This will depend on the residual stresses which result from cooling of the section after rolling or welding. Geometrical imperfections such as initial out-of-straightness also have an effect, and theoreticians have traditionally found it convenient to formulate buckling curves as if nothing else mattered, using a single 'equivalent' geometrical imperfection. Eurocode 3 'imperfection factors' of Table 6.3, each associated with a curve of the set (labelled **a** to **d** in order of severity) represent a continuation of this tradition. For practical purposes, however, the means by which the curves were generated is almost irrelevant; they might as well be regarded as empirical because the assignment of different curves to different sections is made by reference to test results. It is found that deeper-proportioned sections need to be treated more severely than shallow ones and the effects of residual stresses are more severe in welded sections than in rolled ones.

These differences are most pronounced over the middle part of the slenderness spectrum, the one occupied by most practical beams. At very high slenderness, where there is little influence of material strength, the curves tend to converge with each other (and with elastic theory). At very low slenderness, LTB tests indicate that a 'plateau' of $\chi_{LT} = 1$ extends as far as NDS = 0.4 or thereabouts. One of the differences between LTB and regular strut buckling is that for the latter the corresponding plateau only extends half as far.

### A choice of two sets of buckling curves for LTB

Eurocode 3 offers the designer a choice, for I-sections at least, between two sets of curves. These are presented in clauses 6.3.2.2 and 6.3.2.3

respectively. Sharp-eyed readers will observe that the 6.3.2.2 set is identical to the strut curves of 6.3.1, save for the plateau extension granted in 6.3.2.2(4) which, incidentally, results in a cliff edge discontinuity at NDS = 0.4.

The alternative, and generally more productive, set of curves is on offer in 6.3.2.3 for rolled I-sections and equivalent welded I-sections. I-sections obviously includes H-sections, and the word 'equivalent' is interpreted to mean 'of similar dimensions', i e up to about 1 metre in height. But that interpretation hardly matters to UK designers, since the UK National Annex takes advantage of an opportunity to exercise national choice. It does so by resetting the parameters in the buckling curve formulation with the effect that for welded sections the curves of 6.3.2.3 now match those of 6.3.2.2 exactly, and they are denied the plateau extension (in either case). For a welded section, the only remaining advantage to 6.3.2.3 is the availability of a favourable modification factor in 6.3.2.3(2), discussed later.

More positively, the UK NA extends the scope of 6.3.2.3 beyond I-sections to include angles (for moments in the 'major principal plane'), all other hot-rolled sections and cold-formed hollow sections. So while choice is (virtually) taken away for welded sections, it is extended to other sections that are rolled.

Figure 1 compares all the buckling curves from clauses 6.3.2.2 and 6.3.2.3. These are the curves available to rolled sections in the UK.

Two traps for the unwary:

- The only thing the two curves named '**c**' have in common is the 'imperfection factor' used to derive them. Curve **c** of 6.3.2.3 lies well above its 6.3.2.2 namesake (and even exceeds curve **a** of 6.3.2.2 over parts of the slenderness range). It is a similar story for curves **b** and **d**. Curve **a** is not used in 6.3.2.3.
- This apparent advantage of choosing 6.3.2.3 is reduced, though mostly not eliminated, by the differences between Tables 6.4 and 6.5 which assign curves to sections. For example, for a UKC section the choice is between curve **a** of 6.3.2.2 and curve **b** of 6.3.2.3, as shown in Figure 2, and it will remain advantageous to opt for the latter. Similarly for most UKB sections curve **c** of 6.3.2.3 will be preferable to curve **b** of 6.3.2.2, as shown in Figure 3, but beware that the UK NA, with its replacement for Table 6.5, has penalized what we might describe as 'slimline' sections with $h/b$ >3.1 (which includes several of the recent additions to the UKB range) by assigning them to curve **d** of 6.3.2.3. Depending on slenderness, it may be advantageous to choose curve **b** of 6.3.2.2 for these, as shown in Figure 4 (over page).

Another consequence of the UK NA's intervention to replace Table 6.5 is that distinctions are made between monosymmetric and doubly symmetric sections.

Monosymmetric rolled sections, such as Corus ASB, are allowed to use 6.3.2.3, but they too are relegated to curve **d**. However Table 6.4, unaltered by the NA, assigns a rolled I-section with $h/b \leq 2$ to curve **a** of 6.3.2.2 which is likely to be advantageous, as shown in Figure 5 (over page). A welded lookalike, excluded from 6.3.2.3, would be assigned to curve **c** of 6.3.2.2.

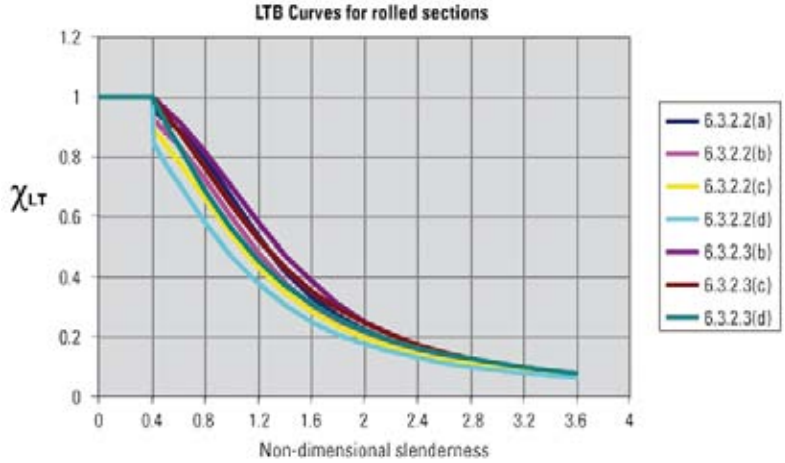


*Figure 1. All seven LTB curves from EC3*

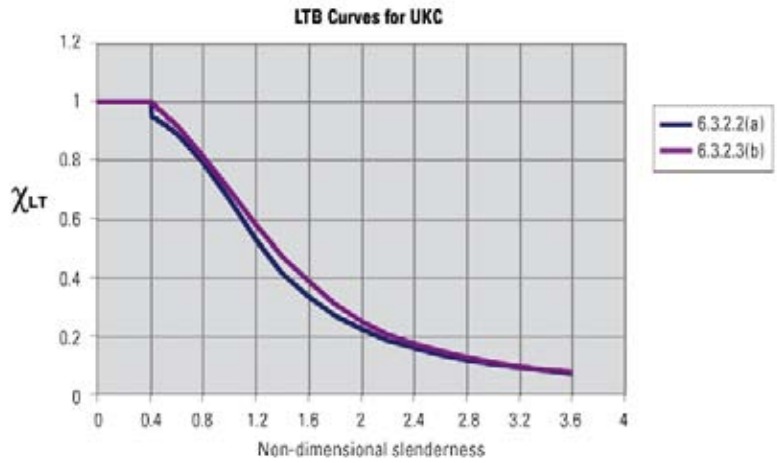


*Figure 2 Choice of curve for UKC ($h/b \leq 2$)*

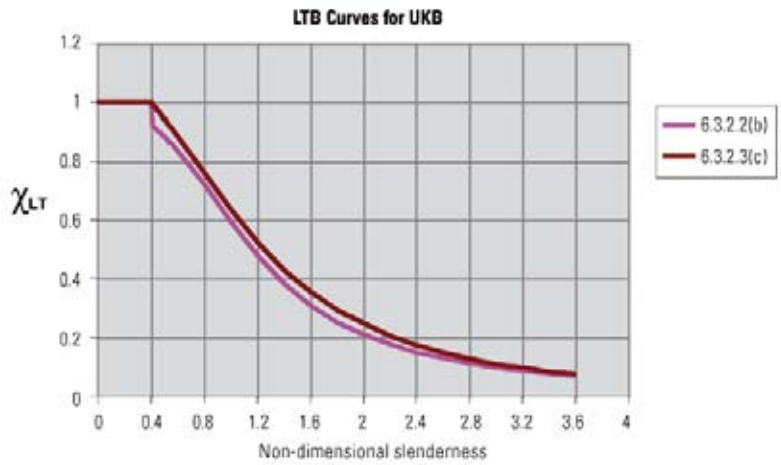


*Figure 3 Choice of curve for most (but not all) UKB ($2 < h/b \leq 3.1$)*

## Welded sections

As already noted, welded beams are relatively harshly treated by the UK NA, so it may become commercially important to consider whether sections such as SFB and Cellbeams could

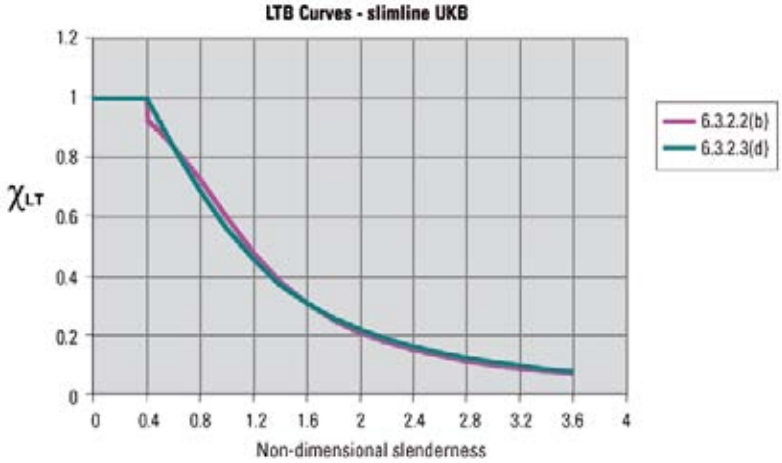


*Figure 4 Choice for 'slimline' (h/b > 3.1) UKB 1016x305, 610x178, 533x165*

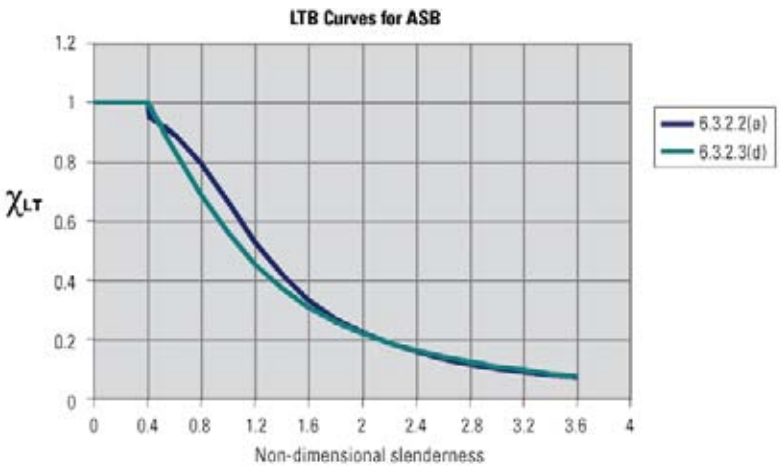


*Figure 5 Choice of curve for ASB*

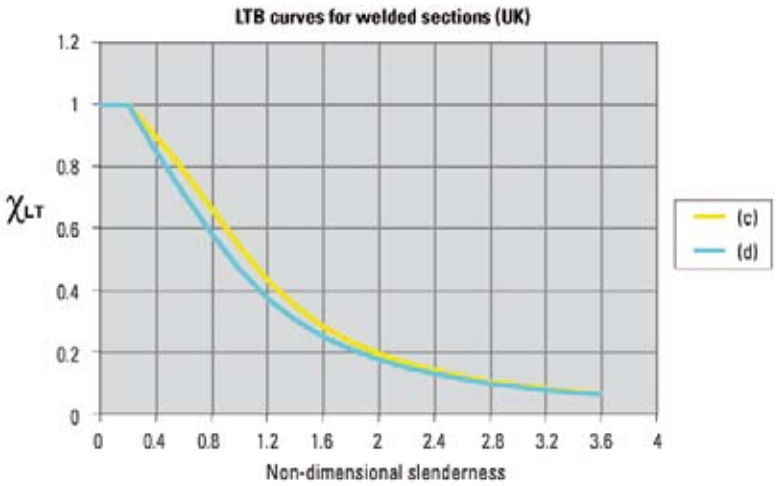


*Figure 6 Curves for welded sections (UK)*

receive honorary 'rolled' status for LTB purposes. Although these are fabricated by welding, it may be argued that the welds are distant enough from the compression flange not to impact the LTB resistance. For these and other out-of-the-ordinary sections, there may be potential for curve assignations to be upgraded if tests are undertaken to overcome the present paucity of data.

The curves for welded sections are shown in Figure 6. There is no choice; the higher curve **c** is for $h/b \leq 2$ and the lower curve d is for typical beam proportions. The UK NA's upper limit (of 3.1) to *h/b* for the use of curve **d** of 6.3.2.3 does not apply to 6.3.2.2, so its effect is a subtle one: purely to deny a slimline welded beam the use of the modification factor.

### A word about the modification factor of 6.3.2.3(2)

This modification factor is to allow for the favourable effect of nonuniform moment, and is influenced by NDS. It can only increase $\chi_{LT}$, so it could safely be ignored. Its favourable effect may not be all that great – a few % at most for a simply supported beam with UDL – but in other situations it can be more significant. Designers should not be perturbed by the fact that nonuniformity of moment may already have been taken into account in the assessment of $M_{cr}$ – this second correction is perfectly in order (as will be a third, if an evaluation of the *k*-factors of expressions 6.61/62 is to follow). It may also be helpful to be reassured that what the UK NA has to say about the modification factor (in NA 2.18) is not in any way conflicting with what is in the Code, merely offering a more general formulation.

The effect of this modification factor could tip the balance in favour of 6.3.2.3 over 6.3.2.2 in a marginal case, because (whether by accident or design) it is not available to the latter.

### Conclusion

While 6.3.2.3's buckling curves (which are specifically for LTB) will normally be more productive than those of 6.3.2.2 (which are, essentially, the strut curves and regarded as a safe lower bound) there are circumstances in which either the Code or the UK NA rules them out. There are also sections whose curve assignations by the Code and/or the NA have the effect that the reverse will, or (depending on slenderness) may, be true. If comparison is necessary to decide which is advantageous, it is not enough simply to look at the buckling curves, because of 6.3.2.3's $\chi_{LT}$ -enhancing modification factor.

### Footnote

The Code's formulae for the lateral buckling curves are to be found in expressions 6.56 (for 6.3.2.2) and 6.57 (for 6.3.2.3), using imperfection factors from Table 6.3 in both cases. The UK NA endorses these recommended values, and the curve assignations of Table 6.4 for use with 6.3.2.2. However it exercises national choice in two ways where 6.3.2.3 is concerned. For welded sections only, it resets the parameters of expression 6.57 so as to make the curves identical to those of 6.3.2.2 (and the strut curves). One of these parameters is the plateau length. Because 6.3.2.2(4) obtains this from 6.3.2.3, the effect is to deny welded beams the plateau extension irrespective of clause allegiance. The second major intervention of the UK NA is its Table 6.5 substitute whose effects, on both rolled and welded sections, are described above.