# Lateral torsional buckling and slenderness

*The expression for slenderness used in the lateral torsional buckling checks given in BSEN1993-1-1:2005 is different to that given in BS5950-1:2000. Mary Brettle, Senior Engineer at the Steel Construction Institute, examines lateral torsional buckling and shows how both slenderness expressions are based on the same elastic critical moment theory.*

## 1. WHAT IS LATERAL TORSIONAL BUCKLING?

Lateral torsional buckling may occur in an unrestrained beam. A beam is considered to be unrestrained when its compression flange is free to displace laterally and rotate. When an applied load causes both lateral displacement and twisting of a member lateral torsional buckling has occurred. *Figure 1* shows the lateral displacement and twisting experienced by a beam when lateral torsional buckling occurs.

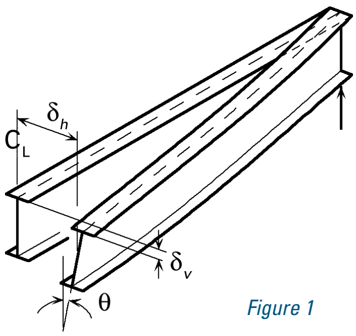

*Figure 1*

### 1.1 What causes the lateral deflection?

The applied vertical load results in compression and tension in the flanges of the section. The compression flange tries to deflect laterally away from its original position, whereas the tension flange tries to keep the member straight. The lateral movement of the flanges is shown in *Figure 2.*

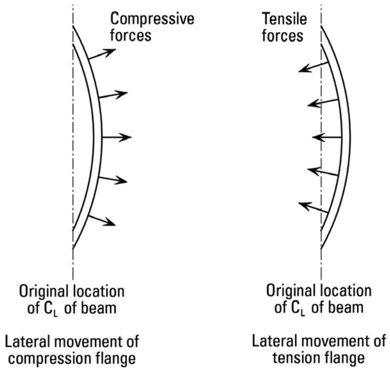

*Figure 2*

The lateral bending of the section creates restoring forces that oppose the movement because the section wants to remain straight. These restoring forces are not large enough to stop the section from deflecting laterally, but together with the lateral component of the tensile forces, they determine the buckling resistance of the beam.

### 1.2 Torsional effect

In addition to the lateral movement of the section the forces within the flanges cause the section to twist about its longitudinal axis as shown in Figure 3. The twisting is resisted by the torsional stiffness of the section. The torsional stiffness of a section is dominated by the flange thickness. That is why a section with thicker flanges has a larger bending strength ($p_b$) than the same depth of section with thinner flanges. This is why Table 20 of BS5950-1:2000 relates the value of pb to the ratio of depth / flange thickness (*D/T*) and Table 7 of BS449-2:1969 relates the value of elastic critical stress ($C_s$) to D/T.

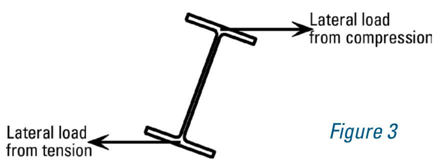

*Figure 3*

### 1.3 What affects lateral torsional buckling

Some factors that influence the lateral torsional buckling behaviour of beams are briefly discussed below:

#### Location of the applied load

The vertical distance between the load application point and the shear centre of the section affects the susceptibility of the section to the effects of lateral torsional buckling. If the load is applied at a location above the shear centre of a section it is more susceptible to lateral torsional buckling than if the load was applied through the shear centre. Applying the load at a location below the shear centre of a section reduces the susceptibility of the section to the effects of lateral torsional buckling. When the load is applied above the shear centre it is known as a destabilising load, with loads applied at or below the shear centre called non-destabilising loads. The effect of a destabilising load is considered by the use of effective lengths given in Table 13 of BS5950-1:2000, where the effective lengths are longer for destabilising loads compared to the non-destabilising loads.

#### The shape of the applied bending moment

The buckling resistance for a section subject to a uniform bending moment distribution along its length is less than the buckling resistance obtained for the same section subjected to a different bending moment distribution. Factors are included in design guidance to allow for the effect of different bending moment distributions. UK designers will be familiar with the use of the equivalent uniform moment factor ($m_{LT}$) in BS5950-1:2000.

#### End support conditions

The end support conditions considered during the development of the basic theory for buckling moments are equivalent to web cleats that stop the web from deflecting laterally and twisting. For end conditions where more restraint is given to the section the buckling moment increases, with the buckling moment decreasing for end supports that offer less restraint to the section. BS5950-1:2000 considers effective lengths when determining the slenderness of a section to account for the effect of end restraint on lateral torsional buckling.

## 2. SECTION SLENDERNESS

The slenderness of a section is used in design checks for lateral torsional buckling. The following factors affect the slenderness of a section:

- Length of the beam
- Lateral bending stiffness of the flanges
- Torsional stiffness of the section

Design codes need to account for the above factors in the guidance they give for determining the slenderness of a section.

The elastic critical moment ($M_{cr}$) is used as the basis for the methods given in design codes for determining the slenderness of a section. The elastic critical moment ($M_{cr}$) is similar to the Euler (flexural) buckling of a strut in that it defines a buckling load. Euler bucking defines the axial compression that will cause a strut to fail in elastic flexural buckling compared with the elastic critical moment that defines the moment that will result in failure due to elastic lateral torsional buckling of a beam. The Elastic critical buckling ($M_{cr}$) and Euler buckling ($P_E$) curves are shown in *Figure 4*.

The buckling moment of a section is affected by plasticity. Therefore the buckling moment resistance ($M_b$) cannot be greater than the plastic moment ($M_{pl}$) of the section. The buckling moment resistance curve shown in *Figure 4* shows that;

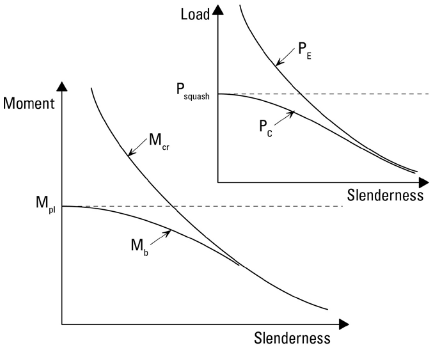

*Figure 4*

- very slender sections fail elastically by excessive lateral torsional buckling at an applied moment close to $M_{cr}$,
- intermediate slender sections fail inelastically by excessive lateral torsional buckling at applied moments less than $M_{cr}$,
- stocky sections will attain their full plastic moment ($M_{pl}$) with negligible lateral torsional buckling.

### 2.1 How British Standards use the elastic critical moment ($M_{cr}$)

British Standard steel design codes all use $M_{cr}$ as the basis for determining the slenderness of a section, but, the expressions used in the codes are not the same. Below the expressions given in some British Standards are considered.

#### BS5950-1:2000

The expression given for uniform I, H and channel sections with equal flanges is: $\lambda_{LT} = uv\lambda\sqrt{\beta_w}$

The above expression does not appear to consider $M_{cr}$. However, when the expressions given in Annex B of BS5950-1:2000 are considered it can be shown how the above expression is based on $M_{cr}$.

$$\lambda_{LT} = \sqrt{\frac{\pi^2 E}{p_E}}$$

Where:

$$p_E = \frac{M_{cr}}{S\beta_w}$$

$\beta_w$ is defined in 4.3.6.9 as $\beta_w = \dfrac{\text{Design Modulus}}{S_x}$ where the design modulus depends on the classification of the cross section. Therefore, the expression for $\lambda_{LT}$ can be rearranged to give:

$$\lambda_{LT} = \sqrt{\frac{\pi^2 E}{p_E}}\sqrt{\frac{M_c}{M_{cr}}}$$

#### BS449-2:1969

The guidance given for bending stresses in plate girders uses the elastic critical stress ($C_s$) to determine the permissible bending stress ($p_{bc}$). The elastic critical stress ($C_s$) can be expressed as:

$$C_s = \frac{M_{cr}}{Z}$$

#### BS5400-3:2000

The guidance given in this British Standard for overall lateral buckling given in clause 9.7.5 explicitly uses $M_{cr}$ as follows:

$$\lambda_{LT} = \sqrt{\frac{\pi^2 E Z_{pe}}{M_{cr}}}$$

Where:

$Z_{pe}$ is defined in 9.7.1 as $Z_{pe} = \dfrac{M_{pe}}{\sigma_{yc}}$

### 3. EUROCODE 3 DESIGN

The lateral torsional buckling design guidance given in BSEN1993-1-1:2005 requires a reduction factor ($\chi_{LT}$) to be applied to the moment resistance of the cross section to give the lateral torsional buckling moment resistance ($M_{b,Rd}$). $\chi_{LT}$ is determined from a factor ($\Phi_{LT}$) and the non-dimensional slenderness factor ($\lambda_{LT}$). The expression given for $\lambda_{LT}$ is:

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$

Where:

$W_y f_y$ is the moment resistance for the section ($M_{b,Rd}$), which is equivalent to $M_c$ in BS5950-1:2000.

### 3.1 Calculating $M_{cr}$

For doubly symmetric sections the expression for $M_{cr}$ is:

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(kL)^2}\left\{\sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 G I_t}{\pi^2 E I_z} + (C_2 z_g)^2} - C_2 z_g\right\}$$

Where

$E$ is the Youngs modulus
$G$ is the shear modulus
$I_z$ is the second moment of area about the minor axis
$I_t$ is the torsion constant
$I_w$ is the warping constant
$L$ is the beam length between points which have lateral restraint
$k$ and $k_w$ are effective length factors
$z_g$ is the distance between the point of load application and the shear centre (see *Figure 5*)
$C_1$ and $C_2$ are coefficients depending on the loading and end restraint conditions.

Further details on the above calculation of $M_{cr}$ can be found in the access Steel document SN003a, which is given on the website: www.accesssteel.com.

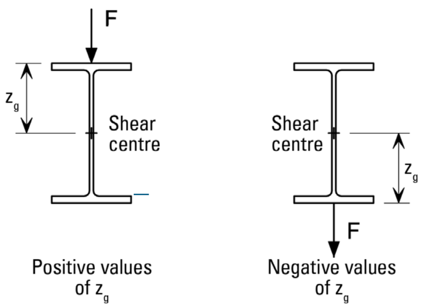

*Figure 5*

### 3.2 Don't panic, you can determine $\lambda_{LT}$ without $M_{cr}$

After seeing the above expression for $M_{cr}$ for doubly symmetric sections designers will be pleased to learn that $\lambda_{LT}$ can be determined without having to calculate $M_{cr}$. From the formula for $M_{cr}$ the following expression for rolled I, H and channel sections with non-destabilising loads has been determined:

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$$\bar{\lambda}_{LT} = \frac{1}{\sqrt{C_1}} 0{,}9\bar{\lambda}_z\sqrt{\beta_W}$$

Where:

$\frac{1}{\sqrt{C_1}}$ is a parameter dependant on the shape of the bending moment diagram, values can be obtained from access Steel document SN002a and the forthcoming SCI / Corus Concise Guide to Eurocode 3.

$$\bar{\lambda}_z = \frac{L}{i_z}\frac{1}{\lambda_1}$$

$L$ is the distance between points of lateral restraint

$i_z$ is the radius of gyration about the minor axis

$$\lambda_1 = 93.9\sqrt{\frac{235}{f_y}}$$

$f_y$ is the yield strength of the steel

$W_y$ is the plastic, elastic or effective section modulus (depending on the section classification)

$W_{pl,y}$ is the plastic section modulus.