Design of Mono-symmetric and Asymmetric Sections in Compression using BS 5950-1:2000

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1. MONO-SYMMETRIC AND ASYMMETRIC SECTIONS IN COMPRESSION

1.1 Introduction

Mono-symmetric and asymmetric sections are all prone to torsional-flexural buckling, as shown in Figures 1 and 2. In many sections, particularly those shown in Figures 1 and 2, torsional-flexural buckling is the critical mode of buckling, even worse than pure minor axis buckling. This article explains how the resistance to torsional-flexural can be calculated using BS 5950-1:2000. An introduction to both torsional and torsional-flexural modes of buckling is given in *Design of Cruciform Sections using BS 5950 1:2000* in the April 2006 issue of New Steel Construction. In that article, both modes of buckling are described and design equations are given for pure torsional buckling of sections in which the centroid coincides with the shear centre.

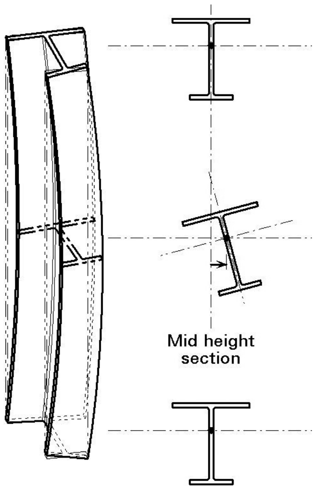

Figure 1. Mono-symmetric section

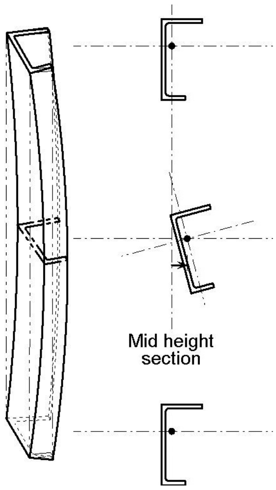

Figure 2. Asymmetric section

The susceptibility to torsional-flexural buckling depends on the position of the centroid of the section relative to the shear centre. Sections in which the shear centre does NOT coincide with the centroid will be prone to torsional-flexural buckling and this should be expected to be the critical mode of buckling for a given effective length. Sections in which the shear centre DOES coincide with the centroid will NOT be prone to torsional-flexural buckling provided that both flanges are effectively restrained at the same points along the member. (In the case of angles, channels and tees, flexural-torsional buckling is effectively allowed for in BS 5950-1 by the slenderness calculations to 4.7.10 and Table 25.)

In sections that are mono-symmetric, the buckling in the plane of symmetry of the section is not affected by torsional components of deflection, as shown in Figure 3. When a mono-symmetric section buckles out of the plane of symmetry as shown in Figure 4, the buckling resistance is less than simple flexural buckling because of the additional torsional components of the deflection.

1.2 Does it matter?

Flexural-torsional buckling will not be critical in bi-symmetric I-sections (eg UCs) with both flanges restrained at the same points along a member. However, where the flanges are not equally restrained, flexural-torsional modes may be the critical modes.

For all mono-symmetric and asymmetric sections, flexural-torsional buckling is likely to produce a mode of buckling lower than the flexural buckling (or strut buckling) modes normally checked by designers.

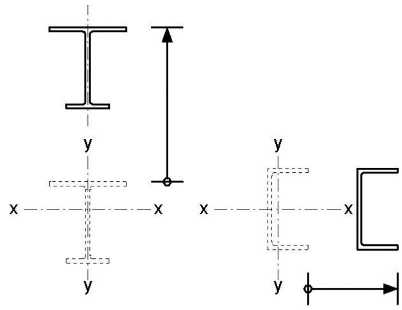

Figure 3 Mono-symmetric sections – buckling in the plane of symmetry

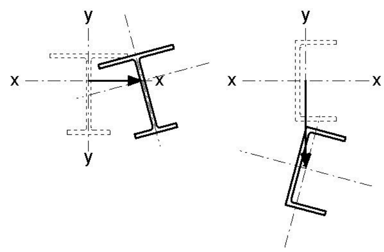

Figure 4 Mono-symmetric sections – buckling out of the plane of symmetry

1.3 What can a designer do?

There are three options open to designers.

The first option is to avoid the problem by using appropriate sections and restraint arrangements. It is simplest to resist compression either by UCs with both flanges equally restrained or by hollow sections. As stated above, these do not suffer from torsional-flexural buckling.

If sections prone to torsional-flexural buckling must be used, then the option of simplifying the calculations should be considered. For example, in the case of an I-section with different flange widths, the section may be thought of as two T-sections with half the web attached to each flange, as shown in Figure 5. Then the lowest compressive strength, p_c, from either of the two T-sections may be taken as the compressive strength for the whole section for buckling out of the plane of the web.

If the problem cannot be avoided, nor the calculations simplified by considering T-sections, the compressive strength for the torsional-flexural mode should be calculated and compared with the compressive strength for flexural buckling. The compression resistance of the member should be calculated using the lowest compressive strength.

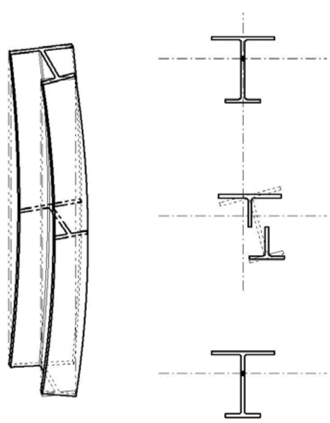

Figure 5 Simplification of the behaviour of a mono-symmetric section as two T-sections

2. COMPRESSIVE STRENGTH

2.1 Use of the elastic critical buckling stress

As described in the previous article, the compressive strength can be calculated from the elastic critical buckling stress, following the procedure in Annex C of BS5950-1:2000. However, it is more convenient to calculate the equivalent slenderness, λ, from the equation in Annex C and then find p_c directly from tables in the Standard.

Annex C gives $p_E = \pi^2 E / \lambda^2$. Therefore, λ can be found from $\lambda = \sqrt{\dfrac{\pi^2 E}{P_E}}$. In the case of torsional-flexural buckling, the value of p_E to be used is the value of p_{ETF} calculated from the equations given below. This allows the designer to use Table 24 of BS 5950-1:2000 to find p_c easily.

2.2 Calculation of the elastic critical buckling stress

The elastic critical load for torsional-flexural buckling can be found in classic references such as *Timoshenko, Strength of Materials, Part 2* or *Timoshenko and Gere, Theory of Elastic Stability*. The elastic critical buckling stress is simply [elastic critical load]/area. The formulae below are written in the symbols in BS 5950-1 but with the BS 5950 1 symbol p_E for the elastic critical stress written to distinguish between four different modes of buckling as follows:

p_{Ex} is for flexural buckling about the x axis,
p_{Ey} is for flexural buckling about the y axis,
p_{ET} is for torsional buckling and
p_{ETF} is for torsional-flexural buckling.

Solving for the general case of torsional-flexural buckling is unpleasantly complicated because it requires the solution of a cubic equation. Solving for the case of mono-symmetric sections is less complicated. Therefore it is convenient to give the solutions for mono-symmetry and for the general case separately.

The axes used in the formulae are the principal axes. In the case of mono-symmetric sections with the plane of symmetry in either the x axis or the y axis, the principal axes are the rectangular axes normally used by designers.

Mono-symmetric sections symmetric about the y axis.

This is the torsional-flexural mode for the common case of I-sections with unequal flanges and symmetrical about the plane of the web, as shown in Figure 1:

$$p_{ETF} = \frac{1}{2B}\left[\left(p_{Ey} + p_{ET}\right) - \sqrt{\left(p_{Ey} + p_{ET}\right)^2 - 4Bp_{Ey}p_{ET}}\right]$$

Mono-symmetric sections symmetric about the x axis

This is the torsional-flexural mode for channel sections with equal flanges:

$$p_{ETF} = \frac{1}{2B}\left[\left(p_{Ex} + p_{ET}\right) - \sqrt{\left(p_{Ex} + p_{ET}\right)^2 - 4Bp_{Ex}p_{ET}}\right]$$

Asymmetric sections

The elastic critical stress, p_{ETF}, must be found by solving the cubic equation:

$$r_0^2\left(p_{ETF} - p_{Ey}\right)\left(p_{ETF} - p_{Ex}\right)\left(p_{ETF} - p_{ET}\right) - p_{ETF}^{\ 2}\, y_0^2\left(p_{ETF} - p_{Ex}\right) - p_{ETF}^{\ 2}\, x_0^2\left(p_{ETF} - p_{Ey}\right) = 0$$

where

$$B = \frac{I_x + I_y}{I_x + I_y + A\left(x_0^2 + y_0^2\right)} = \frac{r_x^2 + r_y^2}{r_x + r_y^2 + x_0^2 + y_0^2}$$ in which,

I_x is the second moment of area about the x axis
I_y is the second moment of area about the y axis
A is the gross area
x_0 is the distance parallel to the x axis between the centroid and the shear centre. In sections symmetric about the y axis, $x_0 = 0$
y_0 is the distance parallel to the y axis between the centroid and the shear centre. In sections symmetric about the x axis, $y_0 = 0$.
r_x is the radius of gyration about the x axis
r_y is the radius of gyration about the y axis
$r_0^2 = r_x^2 + r_y^2 + x_0^2 + y_0^2$

$$p_{Ex} = \frac{\left(\dfrac{\pi^2 EI_x}{L_x^2}\right)}{A} = \frac{\pi^2 E}{(L_x / r_x)^2}$$

$$p_{Ey} = \frac{\left(\dfrac{\pi^2 EI_y}{L_y^2}\right)}{A} = \frac{\pi^2 E}{(L_y / r_y)^2}$$

$$p_{ET} = \frac{1}{I_0}\left(GJ + \frac{n_T^2 \pi^2}{L_T^2} EH\right)$$

where

L_x is the effective length of the member for buckling about the x axis
L_y is the effective length of the member for buckling about the y axis
L_T is the length of the member for torsional buckling
n_T is the number of half-sine waves along the outstands of the member for the torsional buckling mode. For members restrained at both ends, this should be taken as 1.0 unless you are sure you can prove it is greater. For cantilever columns this should not be taken as greater than 0.5.
E is Young's modulus
I_0 is the polar moment of area with respect to the shear centre of the section= $I_x + I_y + A(x_0^2 + y_0^2)$
G is the torsional modulus
J is the torsional constant,
and
H is the warping constant

2.3 Selection of the buckling curve

Where there is torsional-flexural buckling, the buckling involves a component of flexure of the flanges similar to minor axis buckling. Therefore the buckling curve should normally be selected from Table 23 of BS 5950-1:2000 for minor axis buckling.

3. COMBINED AXIAL COMPRESSION AND BENDING

The resistance to combined axial and bending can be checked using the clauses of BS 5950 1:2000 in the normal way, but using the values of compression resistance calculated for torsional-flexural buckling where appropriate. For example, when checking an I-section with unequal flanges as shown in Figure 1 with Clause 4.8.3, the value for P_{cy} should be calculated using the compression strength for torsional flexural buckling if this is less than the value for ordinary flexural buckling about the y axis.

The most common asymmetric beams are I-sections with unequal flanges. BS 5950 1:2000 gives methods to calculate the lateral torsional buckling of these beams in Clause 4.3.6. The main difference from bi-symmetric I-sections is the calculation of "v", which is given in Clause 4.3.6.7. For beams with unequal flanges, this requires the calculation of the flange ratio η.