# P-Delta Analysis

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In last month's edition of *New Steel Construction*, David Brown's article "Multi-storey Frame Design" (ref. 1) explained with great clarity the intended design process for assessing medium sized, orthodox multi-storey frames for sway sensitivity in accordance with BS 5950-1: 2000.

In addition, Alan Rathbone's article "Second-order effects who needs them?" (ref. 2) in the *Structural Engineer* takes the issue one step forward to the more general aspect of why P-Delta effects are important in the context of multi-storey building design.

Both articles focus on multi-storey buildings and talk through the methods used in BS 5950-1: 2000 to account for P-Delta effects, either ignoring them because they are small enough, or including them using the $k_{amp}$ method. This article is aimed primarily at structures that can not be considered as "medium sized orthodox multi-storey buildings" or for structures where $\lambda_{crit} < 4$.

For these structures, an alternative route is available to account for P-Delta effects directly at the analysis stage, thereby incorporating the additional forces and moments required in design.

The objective of this article is to provide engineers with a short introduction to the different methods of P-Delta analysis currently available in commercial software form.

## What are the P-Delta effects?

P-Delta is a non-linear effect that occurs in every structure where elements are subject to axial load. P-Delta is actually only one of many second-order effects. It is a genuine "effect" that is associated with the magnitude of the applied axial load (P) and a displacement (delta).

There are two P-Delta effects:-

- P-"BIG" delta (P-$\Delta$) - a structure effect
- P-"little" delta (P-$\delta$) - a member effect

The magnitude of the P-Delta effect is related to the:-

- magnitude of axial load P
- stiffness/slenderness of the structure as a whole
- slenderness of individual elements.

## Setting the scene

Engineers today typically use linear elastic static (first order) analysis to determine design forces and moments resulting from loads acting on a structure.

First order analysis assumes small deflection behaviour; the resulting forces and moments take no account of the additional effect due to the deformation of the structure under load.

Second-order analysis when accounting for P-Delta combines two effects to reach a solution:-

- large displacement theory - the resulting forces and moments take full account of the effects due to the deformed shape of both the structure and its members.
- "stress stiffening" - the effect of element axial loads on structure stiffness. Tensile loads straighten the geometry of an element thereby stiffening it. Compressive loads accentuate deformation thereby reducing the stiffness of the element.

As structures become ever more slender and less resistant to deformation, the need to consider the P-Delta effect increases. To reflect this, Codes of Practice are referring engineers more and more to the use of second-order analysis in order that P-Delta and "stress stiffening" effects are accounted for, when appropriate, in design. This is as true in concrete and timber design as it is in the design of steelwork.

Engineers have been aware of the P-Delta effects for many years. However, it is only relatively recently that the computational power has become widely available to provide analytical approximations.

In the absence of more rigorous analysis capabilities, in the past, many design codes have incorporated empirical checks and "Good Practice" design guidance to ensure that the magnitude of the P-Delta effect stays within limits for which allowance has inherently been made. Where appropriate, they have made simple and approximate provision for them using methods like amplified sway or extended effective lengths.

While this has ensured safe design, it has perhaps muddied the waters when it comes to developing a clear understanding of the P-Delta "effect" on a structure.

## Methods of accounting for P-Delta effects.

It is possible to spend time and effort avoiding the use of P-Delta analysis by proving that the effects will be negligible but if P-Delta effects are not negligible, which design method should be used?

**Amplified sway** - the most commonly used method, is applicable when $10 < \lambda_{crit} < 4$ . It results in the amplification of some design forces and moments due to the application of a factor $k_{amp}$ to loads causing sway. It is worth saying that sway can be caused by vertical as well as lateral loading. Note - each load combination potentially has its own $k_{amp}$ factor.

**Extended effective lengths** method is approximate and can be awkward to apply to a design

**Second-order analysis** can be used in all situations but must be used if $\lambda_{crit} < 4$. It is worth noting that if the effects are negligible, a P-Delta analysis will return very similar results to those of a linear elastic analysis.

## Is "P-Delta" Analysis the same in all analysis products?

The simple answer is "NO" – the analysis procedures used to determine P-Delta effects vary from one piece of software to the next.

Several methods of accommodating P-Delta effects in analysis have been developed. Some of these methods rely on a constrained problem or set of conditions, and will therefore have documented "limitations"; the critical issue is to understand the differences and be aware of the limitations and conditions.

Remember that analysis models are, by definition, "modelling" of the real condition and hence only provide approximations to the real world.

The P-Delta effect does not distinguish between types or directions of loading. It does not know about floors, floor levels, or the difference between a beam and a column. If the analysis makes assumptions or requires additional data input along these lines, care should be taken to understand and work within the limitations of the analysis.

## Some Analytical P-Delta Methods

Four different analytical methods are considered below. The first two approximate the P-Delta effect using first order elastic analysis, thus care is needed in their use:-

- a "pseudo load" approach
- a "pseudo displacement" approach
- the two-cycle iterative method – accounting for geometric stress stiffness
- non-linear static analysis - full Newton Raphson.

(*Note* – the term "pseudo" indicates that fictitious loads or approximate displacements are applied to the structure to mimic P-Delta behaviour).

## "Pseudo" Approaches

Simple elastic analysis (small displacement) takes no account of this 'secondary' action on the frame. But, it is possible to begin to approximate the P-$\Delta$ effect in two ways using simple elastic analysis.

### *A "Pseudo-Load" Approach*

Typically this sort of approach relies on the structure being subject to predominantly

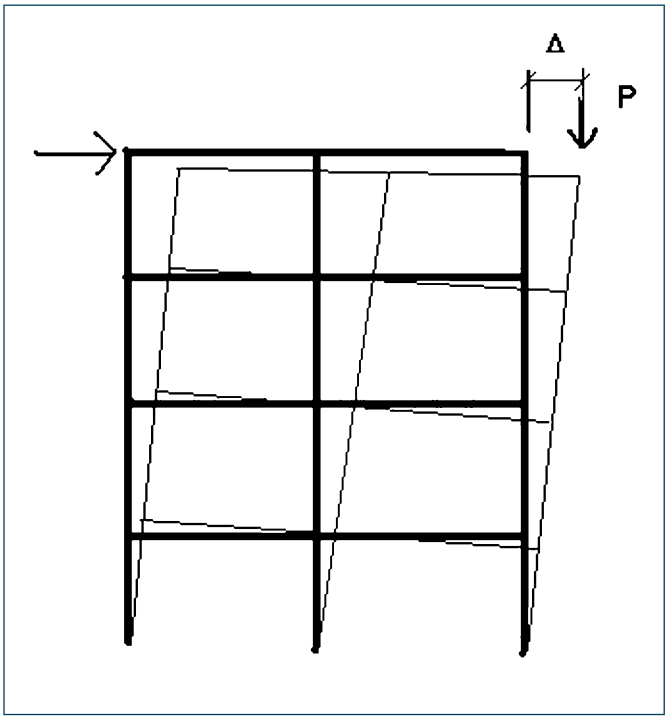


Fig. 1. P-"BIG" delta (P-Δ).

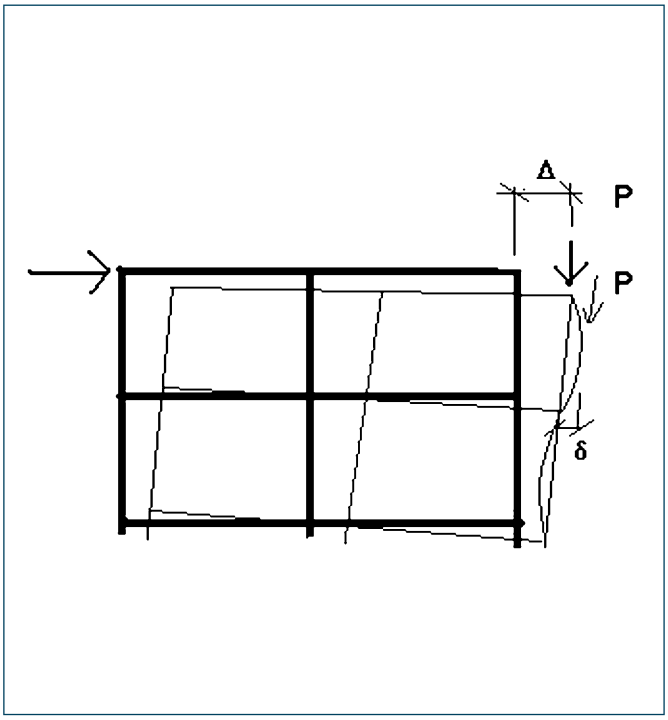


Fig. 2. P-"little" delta (P-δ).

gravity (vertical) loading. Often it also relies on there being defined floor or diaphragm levels within the structure.

An initial analysis allows the "Pseudo" horizontal loads at each floor to be determined. The structure is re-analysed with the "pseudo" loads applied. This process can be repeated (further iteration) with additional adjustment to the pseudo loads.

In the end, the result could be accurate (if the structure fits in with the limitations), but there will be forces within the structure and base reactions that relate to entirely artificial loading.

**REMEMBER** this method does not take account of "stress stiffening", it only deals with one P-Delta effect, P-Δ, it is only relevant to structures that are predominantly gravity loaded, and which have clearly defined floors or diaphragm levels.

### *A "Pseudo-Displacement" approach*

Rather than introducing an artificial load to induce deformation, why not introduce approximate displacements?

An initial elastic analysis is run to establish nodal deflections. The structural model is then re-configured using the deflected geometry and is re-analysed elastically. This process can obviously be repeated progressively but since "stress stiffening" is not taken into account then solutions may not converge.

This approach will not give a good approximation to the P-Delta effect as it takes no account of the 'work done' to move the structure. At best it is a simple approximation. Inaccuracies increase as P-Delta effects increase.

**REMEMBER** this method does not take account of "stress stiffening" and it only deals with one P-Delta effect, P-Δ.

### *The Two Cycle Iterative Method*

The Two-Cycle Iterative Method (Chen and Lui 1991) automates a two-pass analysis procedure during which nodal displacements are used to determine "stress stiffening" in structural elements. The resulting element geometric (stress) stiffness matrix accommodates both the P-Δ and P-$\delta$ effects as well as accounting for "stress stiffness".

Since the geometric (stress) stiffness is used in the method, there are no significant limitations on its use or applicability unless gross deformation occurs when a full non-linear iterative solution is more appropriate.

### *Non-Linear Static (Full Newton Raphson) Analysis*

A full non-linear iterative solution allows for all sorts of non-linear conditions to be accounted for simultaneously, including "stress stiffening" and both the P-Δ and P-$\delta$ effects.

The non-linear solution is carried out in an incremental step-by-step analysis with the total applied loads divided into a number of load steps. The most popular method of solution for non-linear equations is the Newton Raphson method.

When a general "geometric (stress) stiffness matrix" is used in the method, there are no significant limitations on its use or applicability.

## Conclusion

For many years, engineers and Codes of Practice have considered P-Delta effects in structures. In the past this consideration has been simple and limited in application.

Times have changed, Codes of Practice and design tools have moved forwards in recent years. As a result, engineers may have to make informed decisions on the choice of second-order analysis to use. This article is intended to make that choice more informed.

### *References*

1. David Brown, "Multi-storey Frame Design"; New Steel Construction, Vol. 10, No. 6, Nov/Dec 2002.
2. Alan J. Rathbone, "Second-order effects who needs them?", Structural Engineer, Vol. 80, No. 21, Nov. 2002.

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