Joint stiffness calculation

The UK National Annex to BS EN 1993-1-8 discourages the use of numerical methods to calculate joint stiffness, relying on previous satisfactory practice. Despite this, interest in joint stiffness is increasing. Richard Henderson of the SCI illustrates the joint stiffness calculation process set out in the standard and discusses some of the issues.

Introduction

Traditionally, the UK has relied on successful past practice to classify orthodox connections – usually either nominally pinned or nominally rigid. The UK National Annex to BS EN 1993-1-8 endorses that approach and discourages the use of the numerical methods in the standard. The NA also indicates that frame design methods which utilise semi-continuous connection behaviour (the "wind-moment" method, for example) should not use a numerically calculated value, but the connection behaviour should be supported by test evidence or previous satisfactory performance.

Designers are paying increasing attention to connection stiffness, possibly because software is readily available which makes the calculation possible even for unorthodox arrangements. For a limited range of connections, BS EN 1993-1-8 presents a process to calculate the connection stiffness, utilising the same basic connection components which are used to calculate the moment resistance of the joint.

For designers not using software, this article demonstrates the numerical approach given in the standard. The example uses an existing connection design from P398[1], where the basic connection components are already established, shortening the process.

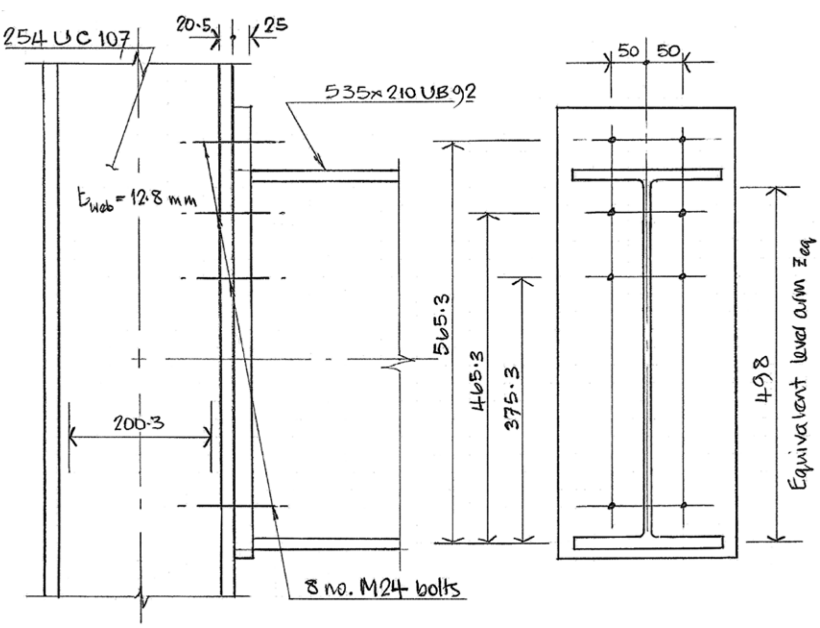

Figure 1: Joint arrangement

Numerical example

Example C2 from the Green Book for moment connections, SCI publication P398, has been used as a convenient bolted beam to column connection to illustrate the method of calculating joint stiffness. According to the UK National Annex to BS EN 1993-1-8, this joint is nominally rigid, simply because it has been designed in accordance with the Green Book.

The expression for the joint stiffness S_j is given in clause 6.3.1(4) as:

$$S_j = \frac{Ez^2}{\mu \Sigma_i \frac{1}{k_i}}$$

where: z is the lever arm defined in para 6.2.7 which depends on the type of joint and the arrangement of the bolts;
μ is the stiffness ratio defined in para 6.3.1(6);
k_i is the stiffness coefficient for basic joint component i.

The stiffness ratio, the ratio of the initial joint stiffness to the stiffness under load, is unity if the applied joint moment $M_{j,Ed}$ is less than 2/3 of the joint resistance $M_{j,Rd}$. For higher moments, the value of μ is given by:

$\mu = (1.5\, M_{j,Ed} / M_{j,Rd})^{\psi}$

The exponent ψ depends on the type of connection and is given in Table 6.8.

Example C2 in P398 is a bolted beam to column joint. The arrangement and member sizes are shown in Figure 1. The moment resistance of the joint is given as 416 kNm.

The relevant stiffness coefficients are identified in Table 6.10 of BS EN 1993-1-8 and for a single sided connection with two or more bolt rows in tension are listed as k_1, k_2 and k_{eq}. Para 6.3.3.1(4) indicates that the equivalent stiffness k_{eq} is based on k_3, k_4, k_5 and k_{10}. The joint components these stiffnesses refer to are given in Table 6.11 in the code and are listed in Table 1.

Stiffness coefficient	Component	Expression
k_1	Column web panel in shear	$0.38A_{VC}/\beta z$; (z_{eq} gives a more accurate value, see Fig 6.15)
k_2	Column web panel in compression	$0.7b_{eff,c,wc}t_{wc}/d_c$; ∞ if stiffened
k_3	Column web panel in tension	$0.7b_{eff,t,wc}t_{wc}/d_c$; ∞ if stiffened
k_4	Column flange bending	$0.9l_{eff}t_{fc}^3/m^3$
k_5	End-plate in bending	$0.9l_{eff}t_p^3/m^3$
k_{10}	Bolts in tension	$1.6A_s/L_b$

Table 1: Relevant stiffness coefficients

▶26

▶24 The quantities are defined in Table 2, taken from the example in P398.

Item	Description	Value
A_{VC}	shear area of column	3810 mm^2
β	transformation parameter (Table 5.4)	1.0
z_{eq}	lever arm	498 mm
b_{eff}, l_{eff}	effective width or length	various
t	component thickness	12.8, 20.5, 25 mm
d_c	clear depth of web	200.3 mm
m	distance of bolt centre to root radius or weld toe	various
A_s	Tensile area of bolt	353 mm^2
L_b	Bolt length	70.5 mm

Table 2: Values of parameters

The first challenge in calculating the stiffness components appears to be the determination of the equivalent lever arm for the column web stiffness coefficient k_1. However, the parameter depends on the effective stiffness for each bolt row r, and the height of the bolt row relative to the centre of compression of the beam flange so the calculation of the effective stiffnesses is in fact the real task. The effective stiffness for each bolt row must be calculated from the stiffness components k_i for that bolt row, given by:

$$k_{eff,j} = \frac{1}{\Sigma_i \frac{1}{k_{i,r}}}$$

The equivalent lever arm is given by:

$$z_{eq} = \frac{\Sigma_r k_{eff,r} h_r^2}{\Sigma_r k_{eff,r} h_r}$$

To complete the list of expressions for stiffness, the equivalent stiffness is given by:

$$k_{eq} = \frac{\Sigma_r k_{eff,r} h_r}{z_{eq}}$$

Using the data from Examples C1 and C2 in P398, the relevant effective widths of plate or lengths of T-stub can be determined. The value corresponds to the effective width or length which gives the lowest resistance for that component in the determination of the resistance of the joint. Where the lowest resistance is for several bolt rows acting as a group, the value for each bolt row is the total length divided by the number of bolt rows in the group, leading to the stiffnesses corresponding to each bolt row. The values are given in Table 3

Stiffness	minimum b_{eff}, l_{eff} (mm)	b_{eff}, l_{eff} (mm)	$k_{i,1}$	$k_{i,2}$	$k_{i,3}$
$k_{3,r}$	$r_1 + r_2 + r_3$	422/3	6.3	6.3	6.3
$k_{4,r}$	$r_1 + r_2 + r_3$	422/3	6.3	6.3	6.3
$k_{5,1}$	r_1	125	30.6	-	-
$k_{5,r}$	$r_2 + r_3$	379/2	-	46.5	46.5
k_{10}	-	-	8.01	8.01	8.01
$k_{eff,r}$	-	-	2.10	2.15	2.15

Table 3: Stiffness values

As an example calculation for the first bolt row,

$$k_{eff,1} = \frac{1}{\frac{1}{6.3} + \frac{1}{6.3} + \frac{1}{30.6} + \frac{1}{8.01}} = 2.10$$

The heights of the bolt rows above the centre of compression are shown in Figure 1 and finally the value of z_{eq} can be determined. The value is:

$$z_{eq} = \frac{1.439 \times 106}{2994} = 498$$

The value for the equivalent stiffness is then:

$$k_{eq} = \frac{2994}{498} = 6.01$$

The remaining stiffnesses can also be calculated and the values are $k_1 = 2.91$ and $k_2 = \infty$ because of the presence of the compression stiffener.

The joint stiffness can now be calculated as follows:

$$S_j = \frac{210 \times (533.1 - 15.6)^2}{\mu\left(\frac{1}{2.91} + 0 + \frac{1}{6.01}\right)} = \frac{102}{\mu} \text{ MNm/radian}$$

The effect of the stiffness ratio μ is shown in Figure 2. For the bolted joint being considered, the value of ψ from Table 6.8 is 2.7. If the design bending moment is greater than two thirds of the bending resistance of the joint, the stiffness is reduced as indicated, to a value of about one third of the maximum stiffness when the applied moment approaches the joint resistance. It should be noted that UK practice is often to optimise the design, so a high utilisation might be expected.

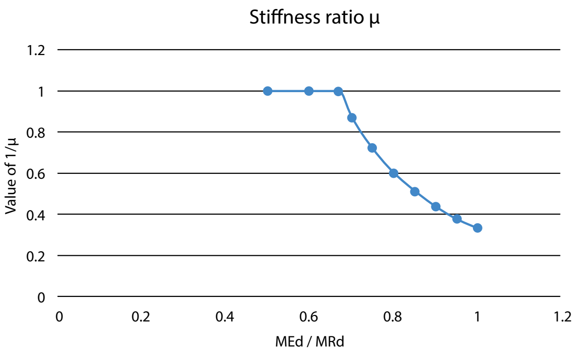

Figure 2: Stiffness ratio, μ

Joint stiffness

Joint classification boundaries on the basis of stiffness are given in clause 5.2.2.5 and Figure 5.4 of BS EN 1993-1-8. The length of the beam and some understanding of the overall frame stiffness is needed, so some assumptions must be made. With reference to Figure 5.4, assuming a 9 m long beam and $k_b = 8$ (for frames with bracing), the requirement for the rigid classification is then $S_{j,ini} \geq k_b EI_b / L_b$.

Substituting values, $k_b EI_b / L_b = 8 \times 210000 \times 55200 \times 10^4 / 9000 = 1.03 \times 10^{11}$ or 103 MNm/radian which is greater than the stiffness calculated in section 2.0, unless $\mu = 1.0$. This assessment would therefore conclude that the joint can only be assumed to be rigid if the design moment is 2/3 of the bending resistance of the joint, or smaller. For unbraced "other frames" where the beams are at least 10 times as stiff as the columns, $k_b = 25$. So for the rigid classification, the initial stiffness must be at least 322 MNm /radian so the joint would be classified as semi-rigid.

Effects of joint flexibility

BS EN 1993-1-1 clause 5.1.2(1) allows the analysis assumption of perfectly pinned or perfectly rigid, as long as the real joint behaviour does not have a 'significant' effect. As an illustration of the effects of the joint stiffness, the same beam was modelled using finite elements with rotational springs at the supports with stiffness equal to the maximum value calculated. The model is unrepresentative because no columns are included in the model. A 9 m span beam is assumed with a uniform load of 41.1 kN/m, giving a free bending moment of 416 kNm. The choice of load is arbitrary. From classical beam theory, a beam with encastré ends will have a support moment of 2/3 of the free bending moment ie 277 kNm and a mid-span moment of 139 kNm. The mid-span deflection will be 1/5 of the simply supported deflection, calculated to be 30.5 mm due to bending alone (no shear deflection). In a braced frame the joint detailed above can be classified as rigid when carrying a design bending moment of 277 kNm or less.

The FE analysis results give a support moment of 130 kNm and a mid-span moment of 286 kNm, with a maximum deflection (including shear deflection) of 20.9 mm. The support moment is about 47% of the encastré value and the deflection 3.4 times the encastré value. The introduction into an analysis model of joint stiffnesses calculated using BS EN 1993-1-8, although classified as "rigid" clearly has a profound effect on the behaviour of the structure and a decision to adopt a structural scheme that relied on frame stiffness and bolted beam to column joints would need to be considered carefully. The "wind-moment" method was shown to be adequate by frame analysis incorporating connection stiffness demonstrated by test, thus meeting the requirements of the UK National Annex.

Traditional approaches to unbraced frame deflection calculations have assumed that joints are rigid and deformation of the members is the source of overall building deflections, unless joints between members are of significant size relative to the member lengths. Such assumptions may need to be reconsidered for certain structures.

Conclusions

If joint stiffness is to be considered at all:

1) The manual calculation of stiffness is very laborious and it would be unrealistic to try to design a real structure in this way. Design software to calculate the joint stiffness is essential for projects of any significant size.
2) The sequence of design and sizing is likely to be iterative because the joint arrangements could affect both the serviceability and strength limit states.
3) Flexibility of bolted end-plate joints in beam to column connections in unbraced frame structures could have significant effects on the stability of the structure.

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