Buckling resistance of uniform members in bending

Richard Henderson of the Steel Construction Institute discusses the phenomenon of lateral-torsional buckling.

Introduction

A grid of beams is usually divided into primary and secondary beams and where there is no floor slab to provide continuous support to the compression flanges, the secondary beams provide discrete restraints to the primary. An end plate connection to the primary beam web detailed in accordance with the Green Book rules may be considered to provide a fork end restraint. The secondary beams also apply point loads to the primary and, for this type of connection, the loads are not destabilizing. The system of point loads results in a shear force diagram for the primary beam with constant values between the point loads and a bending moment diagram made up of straight lines (ignoring the effect of the primary beam self-weight).

In determining the resistance of the beam to bending, especially in hand calculations, it is common to consider the primary beam in segments defined by the incoming secondary beams where the segments have defined end restraints and end moments taken from the bending moment diagram of the full beam. This approach corresponds to the conditions set out in clause 6.3.3 of Eurocode 3 which deals with uniform members in bending and axial compression and the effect of these two actions in combination. Note 1 to clause 6.3.3(2) states: "The interaction formulae are based on the modelling of simply supported single span members with end fork conditions and with or without continuous lateral restraints, which are subjected to compression force, end moments and/or transverse loads". Taking the segments one by one is usually on the safe side as the study described in the following sections shows. The purpose of the study is to determine what effect continuity of the beam beyond the segment being considered has on the beam's calculated bending resistance.

Beam	Length (m)	No of point loads / restraints	Segment length (m)						
			1	2	3	4	5	6	7
1	9	2	3	3	3				
2	9	2	3.5	3	2.5				
3	15	4	3	3	3	3	3		
4	15	4	3.5	2.5	3	3.5	2.5		
5	15	6	2	2	2	3	2	2	2
6	15	4	3.5	2.5	3	2.5	3		

Table 1: Arrangement of beams and beam segments

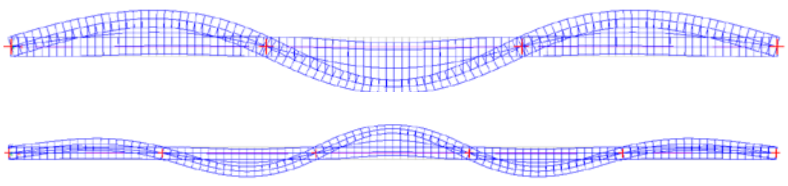

Figure 2: Buckled shape: 3-segment and 5-segment beams

Beams studied

A series of loading arrangements on a 610 × 229 UB 140 was examined. All the arrangements were chosen to result in a 3 m segment of beam subject to a uniform moment of 1200 kNm. The point loads were always applied at restraint positions and beams of length 9 m and 15 m were considered. The loads and restraint positions were chosen such that the lengths of the segments were not always the same so that the half-wave lengths of the buckled shape were uneven. The arrangements are set out in Table 1.

As an illustration, the bending moment diagrams for beams 2 and 6 (neglecting the beam self weight) are shown in Figure 1.

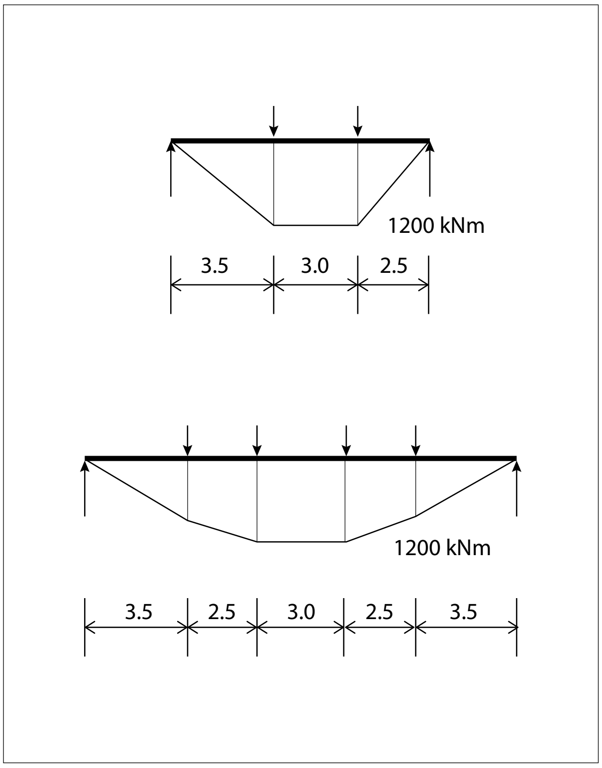

Figure 1: Bending moment diagrams, beams 2 and 6

Beams 1 and 3 have equally spaced loads and restraints, forming segments 3 m long. The buckled shape of the beam calculated by LTBeamN in determining M_{cr} is shown in plan in Figure 2. The top compression flange buckles into a series of half-waves. In each case, the central segment has a uniform bending moment and the adjacent segments have either triangular or trapezoidal-shaped bending moment diagrams. The amplitude of the half-waves can be seen to reduce where the bending moment is not uniform.

▶28

▶26 Where the bending moment is uniform over the whole beam, the half-waves of the buckled shape can be seen to have the same amplitude as shown in Figure 3.

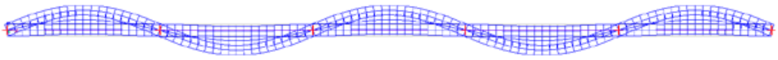

Figure 3: Buckled shape: 5-segment beam, uniform moment

Beam	Segment	length (m)	method	M_{cr} (kNm)	M_{cru} (kNm)	unity factor
1	1	3.0	Blue Book	-	-	0.839
1	2	3.0	Blue Book	-	-	0.984
1	1	3.0	hand calc.	5964	3370	0.840
1	2	3.0	hand calc.	3370	3370	0.982
1	1	3.0	LTBeamN	6235	3366	0.840
1	2	3.0	LTBeamN	3365	3366	0.982
1	-	9.0	LTBeamN	4559	3366	0.866
2	1	3.5	LTBeamN	4709	2544	0.840
2	2	3.0	LTBeamN	3366	3366	0.982
2	3	2.5	LTBeamN	8759	4725	0.852
2	-	9.0	LTBeamN	4636	3193	0.841
3	2	3.0	LTBeamN	4029	3366	0.908
3	3	3.0	LTBeamN	3366	3366	0.982
3	-	15.0	LTBeamN	4263	3366	0.888
4	2	2.5	LTBeamN	5519	4729	0.867
4	3	3.0	LTBeamN	3366	3366	0.982
4	4	3.5	LTBeamN	3206	2544	0.941
4	-	15.0	LTBeamN	4251	3234	0.882
5	3	2.0	LTBeamN	7877	7223	0.840
5	4	3.0	LTBeamN	3366	3366	0.982
5	-	15.0	LTBeamN	6003	3365	0.840
6	2	2.5	LTBeamN	5430	4725	0.872
6	3	3.0	LTBeamN	3366	3366	0.982
6	-	15.0	LTBeamN	4725	3227	0.848

Table 2: Analysis results

Beam Resistances

The resistances of beam segments and beams identified in Table 1 have been calculated for comparison. The segments examined all have a maximum bending moment of 1200 kNm with a bending moment diagram which is either uniform or trapezoidal, except for the 9 m long beams where the bending moment diagram is triangular in the non-uniform moment segments.

The resistances have been determined using EC3 clause 6.3.2.3 for rolled section with the modified strength reduction factor $\chi_{LT,mod}$ from 6.3.2.5(2) and the UK National Annex. The correction factor k_c is determined from the C_1 factor where

$$C_1 = \frac{M_{cr}}{M_{cru}} \quad and \quad k_c = \frac{1}{\sqrt{C_1}}$$

M_{cru} is the elastic critical moment for a uniform moment on the segment. For interest, the unity factors are calculated for Beam 1 using the Blue Book method, by hand and by using LTBeamN to determine values of the critical moments. In addition to considering beam segments defined by the fork-end restraints, LTBeamN was used to analyse the whole beam and determine the critical moments for this case. The results are presented in Table 2.

For beam 1, the Blue Book, hand and LTBeamN methods reassuringly give unity factors which vary by 0.2%. The Blue Book approach probably differs from the other two because the tabulated values in the Book use 3 significant figures. All the 3 m long segments in the beams examined where the bending moment is uniform and equal to 1200 kNm are essentially the same with a unity factor of 0.982.

A closer examination of the results for the full length beams shows that beam 5 has the lowest unity factor of 0.840, about 85% of 0.982. The reduction in unity factor is due to the effect of the continuity of the beam on either side of the segment carrying the uniform bending moment; the continuity is obviously not present if the segments are considered alone. All the beams exhibit this effect to varying degrees. The spacings of restraints in beam 5 have been chosen to inhibit the twisting of the segment with the uniform moment as much as possible. A plan view of the buckled shape of beam 5 is shown in Figure 4.

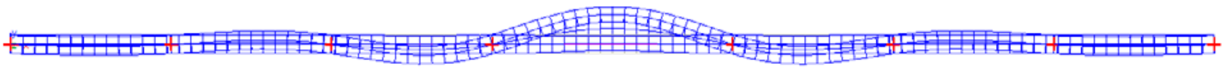

Figure 4: Beam 5 buckled shape

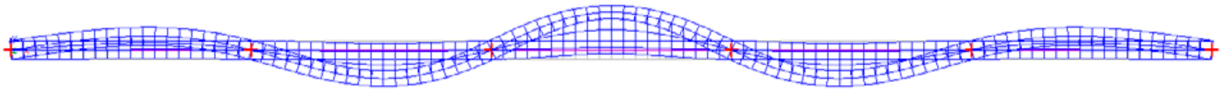

Figure 5: Beam 3 buckled shape

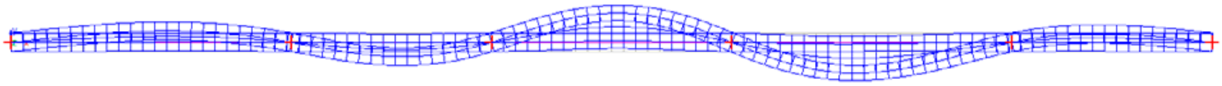

Figure 6: Beam 4 buckled shape

To illustrate the effect of continuity, the restraints are spaced at 2 m apart (except at the central segment), which may be considered unrealistically close spacing for secondary beams.

Beam 3 exhibits the highest unity factor, equal to 0.888 indicating that the continuity has the least effect. The spacing of the restraints are all equal at 3 m, allowing equal length half-waves. The buckled shape is shown in Figure 5.

The next highest unity factor 0.882 for Beam 4. The longer segment next to the segment with uniform moment allows a greater amplitude of lateral torsional distortion in the uniform moment segment. The buckled shape is shown in Figure 6

Conclusion

For the beams examined, continuity of the element beyond the most highly loaded segment (that with a uniform bending moment of 1200 kNm) results in a lower unity factor than is exhibited when considering individual beam segments. For beam 5, the unity factor is reduced from 0.982 to 0.840, 85% of the value for the individual segment. The lower unity factor corresponds to a higher buckling resistance moment $M_{b,Rd}$ for the beam. For the cases where the secondary beam spacing is equal, the corresponding unity factors are 0.866 for a 9 m beam with two point loads and 0.888 for a 15 m beam with four point loads. The buckling resistance moments are calculated as 1351 kNm and 1385 kNm respectively, compared with 1220 kNm for the individual segment. Considering individual segments can therefore be seen to be on the safe side for all the arrangements considered and if extra resistance has to be squeezed out of an existing beam designed segment by segment because of a change in circumstances, an extra 10% could possibly be found by considering the beam as a whole.